Exam from ARO **Name:** ______________________
Login: ______________________
Number of submitted sheets: ______________________

June 15, 2023

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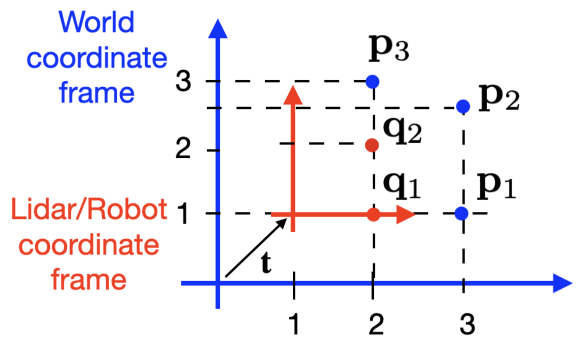
1. **Pose from unknown correspondences:** Consider the problem of estimating the 2D translation of a robot in a 2D world from unknown correspondences; see Figure below. The map consists of three blue points $\mathbf{p}_1 = [3, 1]^\top, \mathbf{p}_2 = [3, 2.5]^\top, \mathbf{p}_3 = [2, 3]^\top$ in the "World coordinate frame" (blue arrows). The robot measures two red points $\mathbf{q}_1 = [1, 0]^\top, \mathbf{q}_2 = [1, 1]^\top$ in its own "Lidar/Robot coordinate frame" (red arrows). The rotation of the rcf is assumed to be fixed and aligned with wcf (i.e. $\mathbf{R} = \mathbf{I}$).



a) (4 points) Given some correspondences $j(i)$ between the measured pointcloud $\mathbf{q}_i$ and the map $\mathbf{p}_{j(i)}$, what is the point-to-point and point-to-plane distance?

$$\rho^{\text{point-point}}(\mathbf{t}) = \sum_i$$

$$\rho^{\text{point-plane}}(\mathbf{t}) = \sum_i$$

(fill in criterions)

b) (2 points) Draw a corresponding spring-like mechanical machine, the equilibrium of which will be the minimizer of point-to-point distance for some correspondences.

c) (2 points) Assume that correspondences are unknown, and the initial guess for the robot translation in the wcf is $\mathbf{t} = [1, 1]^\top$. Perform the first iteration of the Iterative Closest Point (ICP) algorithm with point-to-point distance. What is the resulting translation after the first iteration?

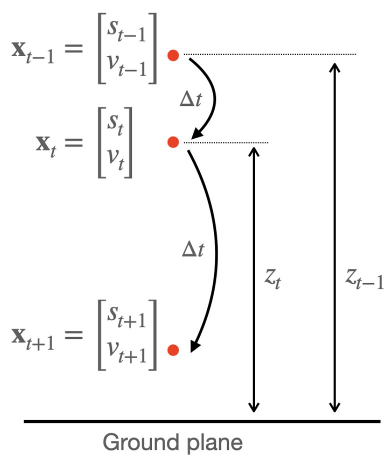
$\begin{bmatrix} \mathbf{t}_x^{\text{1st\_iter}} \\ \mathbf{t}_y^{\text{1st\_iter}} \end{bmatrix} =$ (the answer is a 2-dim vector)

d) (2 points) Given the previously estimated translation, perform the second iteration of the ICP algorithm:

$\begin{bmatrix} \mathbf{t}_x^{\text{2nd\_iter}} \\ \mathbf{t}_y^{\text{2nd\_iter}} \end{bmatrix} =$ (the answer is a 2-dim vector)

2. **Factorgraph localization:**

Consider a space exploration mission during which a robot is deployed above an unknown planet with unknown initial velocity $v_0 \in \mathbb{R}$ and initial height $s_0 \in \mathbb{R}$. The state in time t is 2-dim vector $\mathbf{x}_t = [s_t, v_t]^\top$. The robot has no engine to provide any control; hence the trajectory corresponds to the free fall up to an unmodeled noise. Since the planet is unknown, the gravitational acceleration $g \in \mathbb{R}$ is considered to be also unknown. You want to jointly estimate the gravitational acceleration g and 2-dimensional trajectory of states $\mathbf{x}_0, \ldots, \mathbf{x}_t$.



a) (2 points) The robot is equipped with a height measuring sensor that delivers height measurements $z_t \in \mathbb{R}$ with zero-mean Gaussian noise and unit variance, i.e. $\mu^z = 0, \sigma^z = 1$. What is the measurement probability of this sensor:

$p(z_t|\mathbf{x}_t) =$

b) (2 points) Let's assume that the potential influence of the atmosphere on the free fall is negligible; therefore, the path travelled by the robot over time Δt can be estimated by double integrating the acceleration g over time. Given the previous state $\mathbf{x}_{t-1}$, an acceleration g, and time Δt, what is the current state prediction?

$$\mathbf{x}_t = \begin{bmatrix} s_t \\ v_t \end{bmatrix} =$$

c) (2 points) Assuming that the unmodeled noise of this process is zero-mean Gaussian with unit variance, what is the state transition probability (you can think about the unknown g-acceleration as of static control):

$$p(\mathbf{x}_t|\mathbf{x}_{t-1}, g) =$$

d) (2 points) Derive the maximum likelihood estimate of $\mathbf{x}0, \dots, \mathbf{x}_t, g$, as a function of previously constructed measurement and transition probabilities:

$$\mathbf{x}_0^\star, \dots, \mathbf{x}_t^\star, g^\star = \arg\max_{\mathbf{x}_0,\dots,\mathbf{x}_t,g} p(\mathbf{x}_0, \dots, \mathbf{x}_t, g|z_1, \dots z_t) =$$

e) (1 point) Draw corresponding factor graph with denoted states, actions and measurements and factors.

f) (2 points) Simplify the ML estimate into the optimization-friendly form, such as the (non)-linear least squares problem:

$$\mathbf{x}_0^\star, \ldots, \mathbf{x}_t^\star, g^\star = \operatorname*{arg\,min}_{\mathbf{x}_0, \ldots, \mathbf{x}_t, g} \sum_t$$

g) (1 point) Write down the vector of residuals for unknown variables. What is its dimensionality?

h) (2 points) Sketch a Jacobian for the residuals and fill at least one row with exact expressions. What is the dimensionality of the Jacobian?

3. **Completeness:**

   a) (2 points) You have a path planning problem that has no solution. What answer will be provided by a complete path planning method and why?

4. **Potential field:**

   a) (3 points) Describe (by text/sketch/pseudocode) how planning using Potential field works.

5. **Vertical cell decomposition:**

   a) (3 points) Describe (by text/sketch/pseudocode) how planning using the Vertical cell decomposition works.

   b) (2 points) What is true for this method?

      (a) Stochastic
      (b) Probabilistic complete
      (c) It assumes a point robot
      (d) Complete
      (e) Optimal (from path length point of view)
      (f) Resulting path has always the minimal clearance

6. **RRT***

   a) (2 points) Describe (by text/sketch/pseudocode) how rewiring in RRT* works.

7. **Collision-detection**

   a) (2 points) Describe the principle of hierarchical collision detection.

8. **Narrow passage problem**

   a) (2 points) What is the narrow passage problem? Which methods are related to this problem?

9. **Configuration space** Let's assume a rectangle-shaped robot moving in a 2D workspace. Let $\mathcal{C} = \mathcal{C}_{free} + \mathcal{C}_{obst}$ is the configuration space, and $vol(\mathcal{C}_{obst}) > 0$, where $vol()$ denotes the volume of the set.

   a) (2 points) Consider the case where the robot cannot rotate. What happens if you enlarge the robot?

      - $vol(\mathcal{C}_{obst})$ increases
      - $vol(\mathcal{C}_{obst})$ decreases
      - $vol(\mathcal{C}_{obst})$ remains same
      - $vol(\mathcal{C}_{free})$ increases
      - $vol(\mathcal{C}_{free})$ decreases
      - $vol(\mathcal{C}_{free})$ remains same

   b) (2 points) Consider the case where the robot can rotate. What happens if you enlarge the robot?

      - $vol(\mathcal{C}_{obst})$ increases
      - $vol(\mathcal{C}_{obst})$ decreases
      - $vol(\mathcal{C}_{obst})$ remains same
      - $vol(\mathcal{C}_{free})$ increases
      - $vol(\mathcal{C}_{free})$ decreases
      - $vol(\mathcal{C}_{free})$ remains same