Kalman filter

Karel Zimmermann

Prerequisities: Conditional independence

A - someone is good student

B - full transcript of records

C - average grade

How can you measure it?

$$p(A \mid B, C) = p(A \mid B)$$

What is the natural interpretation?

Prerequisities: Conditional independence

A - someone is good student

B - full transcript of records

C - average grade

How can you measure it?

$$p(A \mid B, C) = p(A \mid B)$$

What is the natural interpretation?

Def: A is *conditionally independent* on C given B iff $p(A \mid B, C) = p(A \mid B)$

Complete states

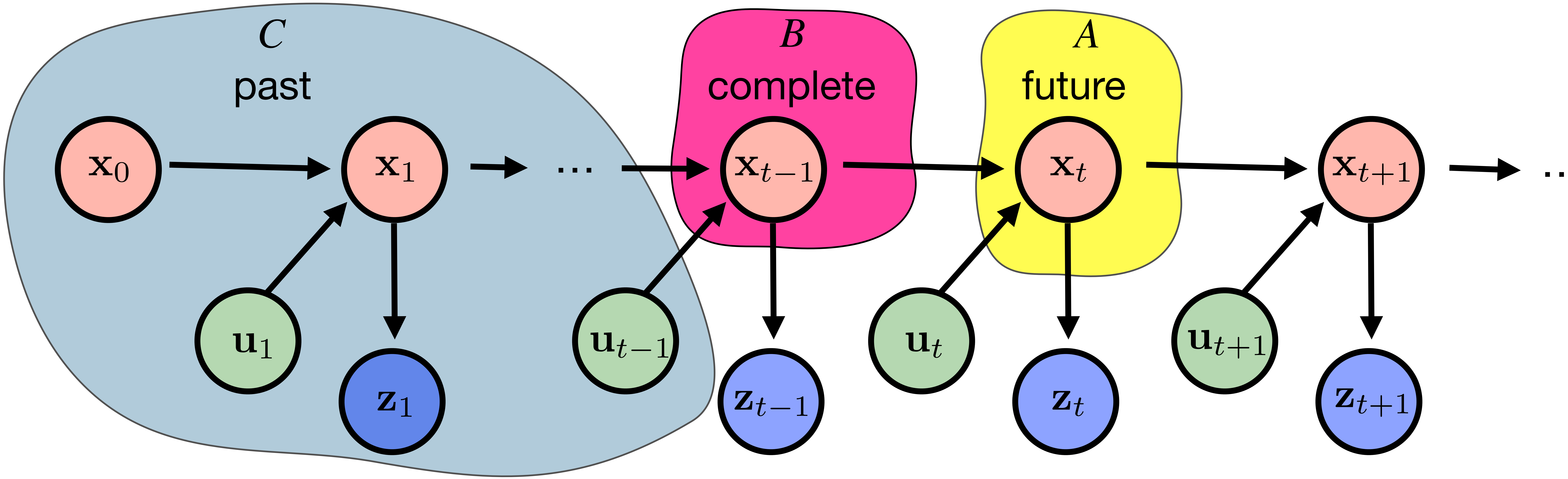
Complete states: $\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_t \in \mathcal{R}^n$

Def: A is *conditionally independent* on C given B iff $p(A \,|\, B, C) = p(A \,|\, B)$

Def: State $\mathbf{x}_{t-1}$ is *complete* iff future $\mathbf{x}_t$ is conditionally independent on past given $\mathbf{x}_{t-1}$

Consequences:

state-transition probability: $p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) = p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t, \mathbf{x}_{0:t-2}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t-1})$



Complete states

Complete states: $\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_t \in \mathcal{R}^n$

<u>Def:</u> A is <u>*conditionally independent*</u> on C given B iff $p(A\,|\,B, C) = p(A\,|\,B)$

<u>Def:</u> State $\mathbf{x}_{t-1}$ is <u>*complete*</u> iff future $\mathbf{x}_t$ is conditionally independent on past given $\mathbf{x}_{t-1}$

<u>Consequences:</u>

state-transition probability: $p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) = p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t, \mathbf{x}_{0:t-2}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t-1})$

measurement probability: $p(\mathbf{z}_t|\mathbf{x}_t) = p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{x}_{0:t-1}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$

Factor graph

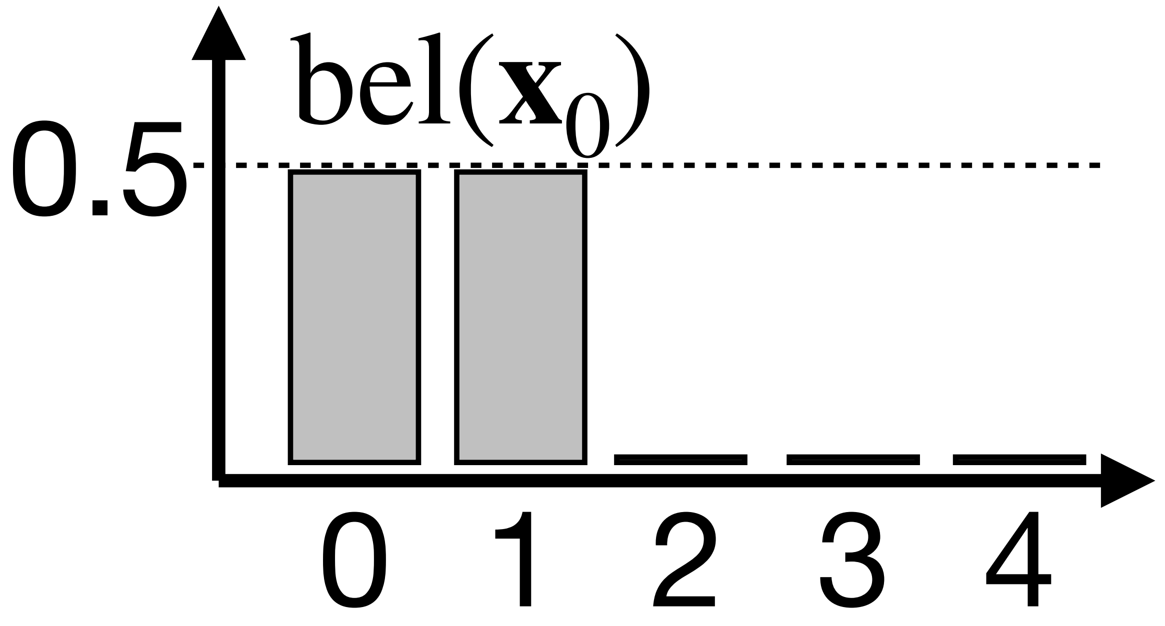
measurement probability: $p(\mathbf{z}_t|\mathbf{x}_t) = p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{x}_{0:t-1}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$

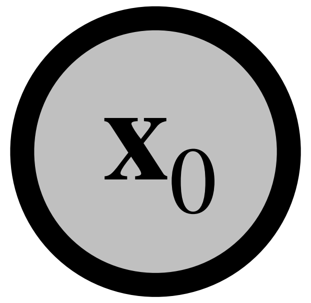
state-transition probability: $p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) = p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t, \mathbf{x}_{0:t-2}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t-1})$

Can I get the optimal $\mathbf{x}_3$ from this factor graph?

Bayes filter

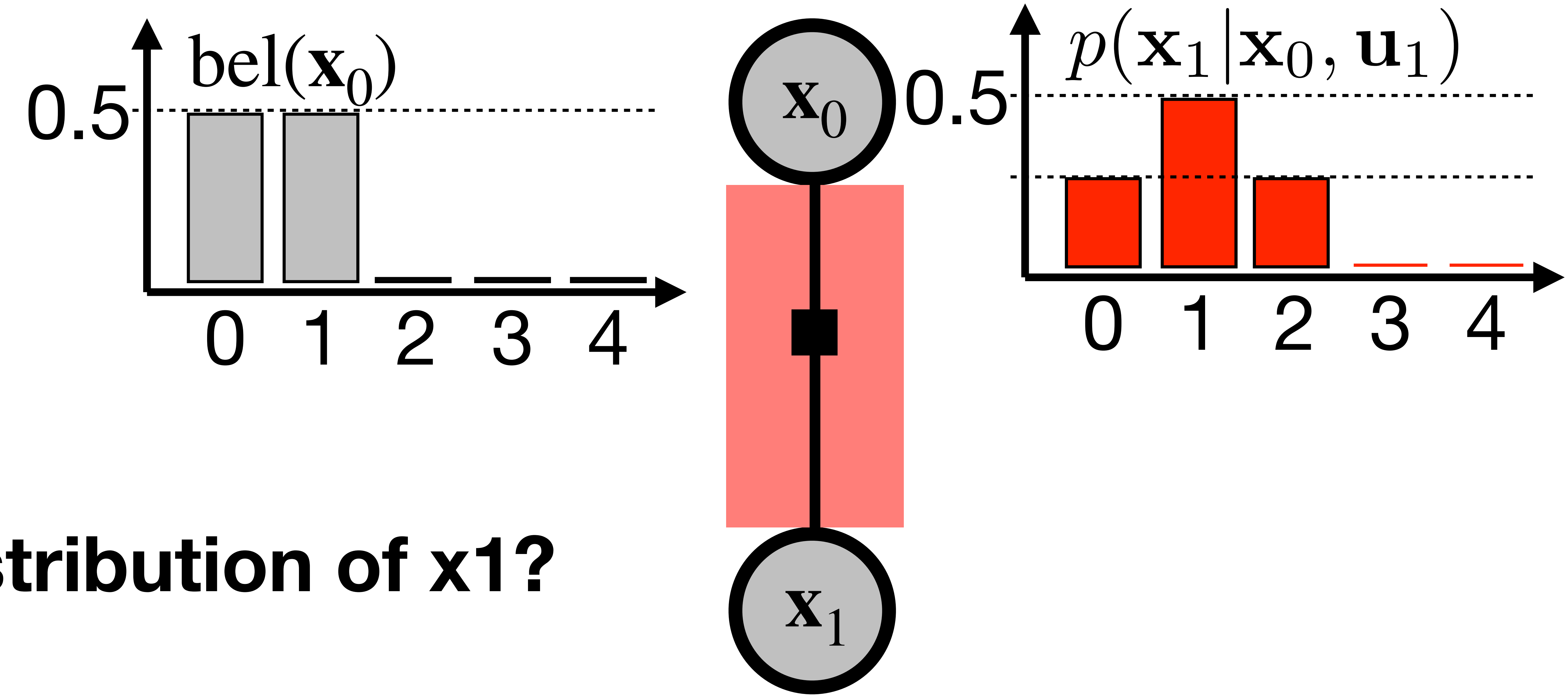
Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$





Bayes filter

Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$



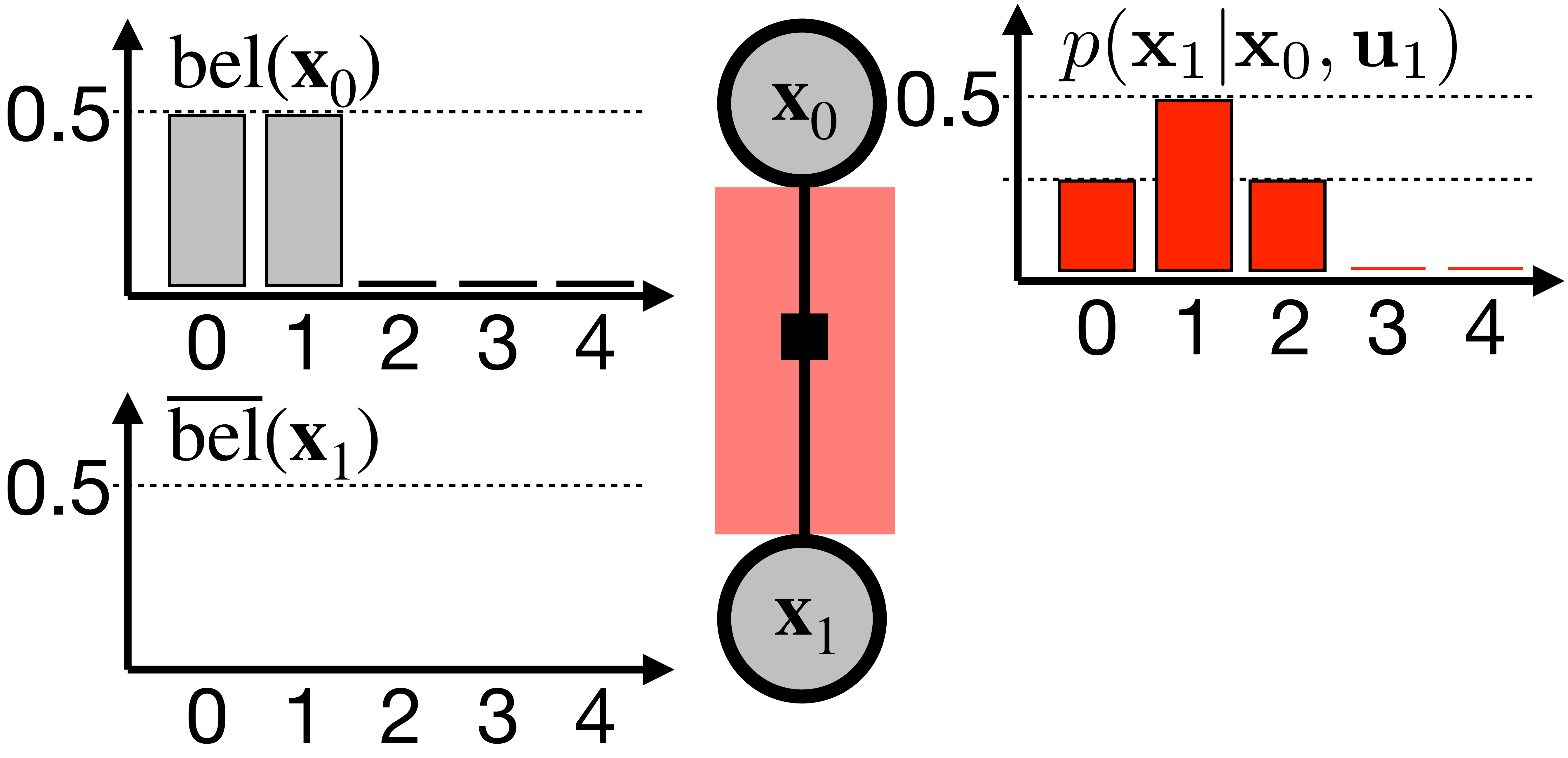
How do you estimate probability distribution of x1?

Bayes filter

Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$

Prediction step (action $\mathbf{u}_t$ performed):

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t)\mathrm{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}$$

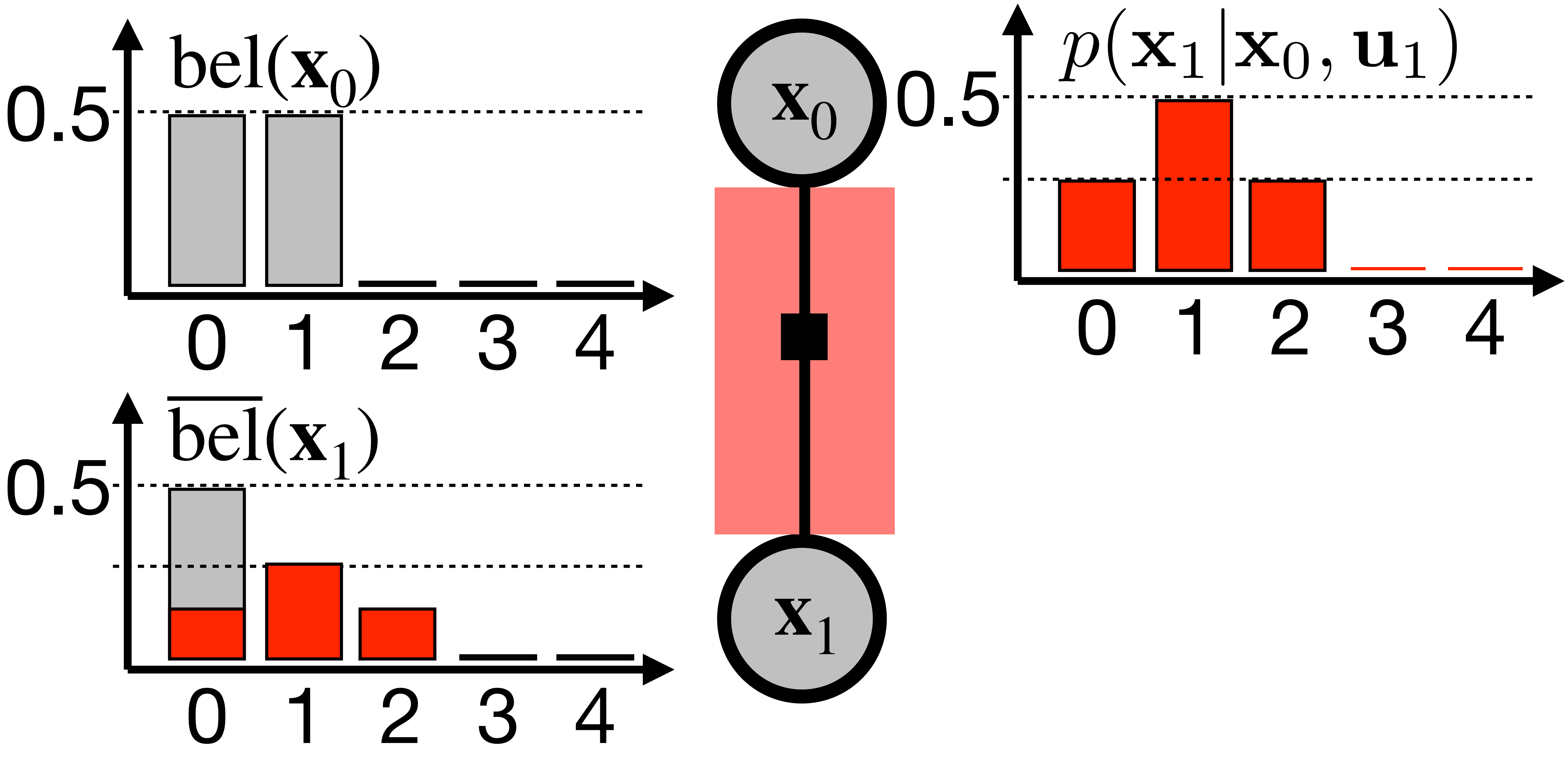


Bayes filter

Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$

Prediction step (action $\mathbf{u}_t$ performed):

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_t) \mathrm{bel}(\mathbf{x}_{t-1}) d\mathbf{x}_{t-1}$$



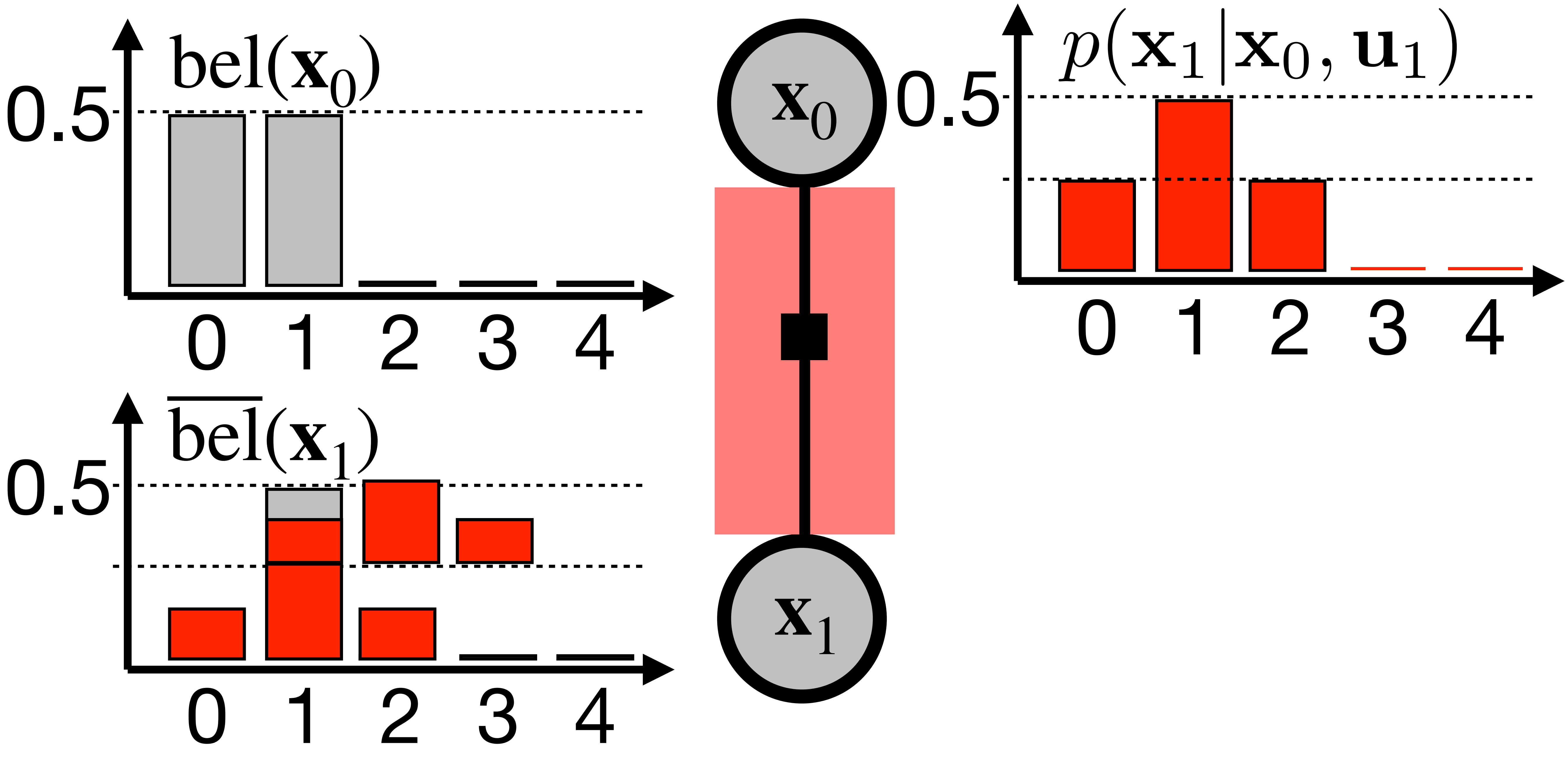
Bayes filter

Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$

Prediction step (action $\mathbf{u}_t$ performed):

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t)\mathrm{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}$$



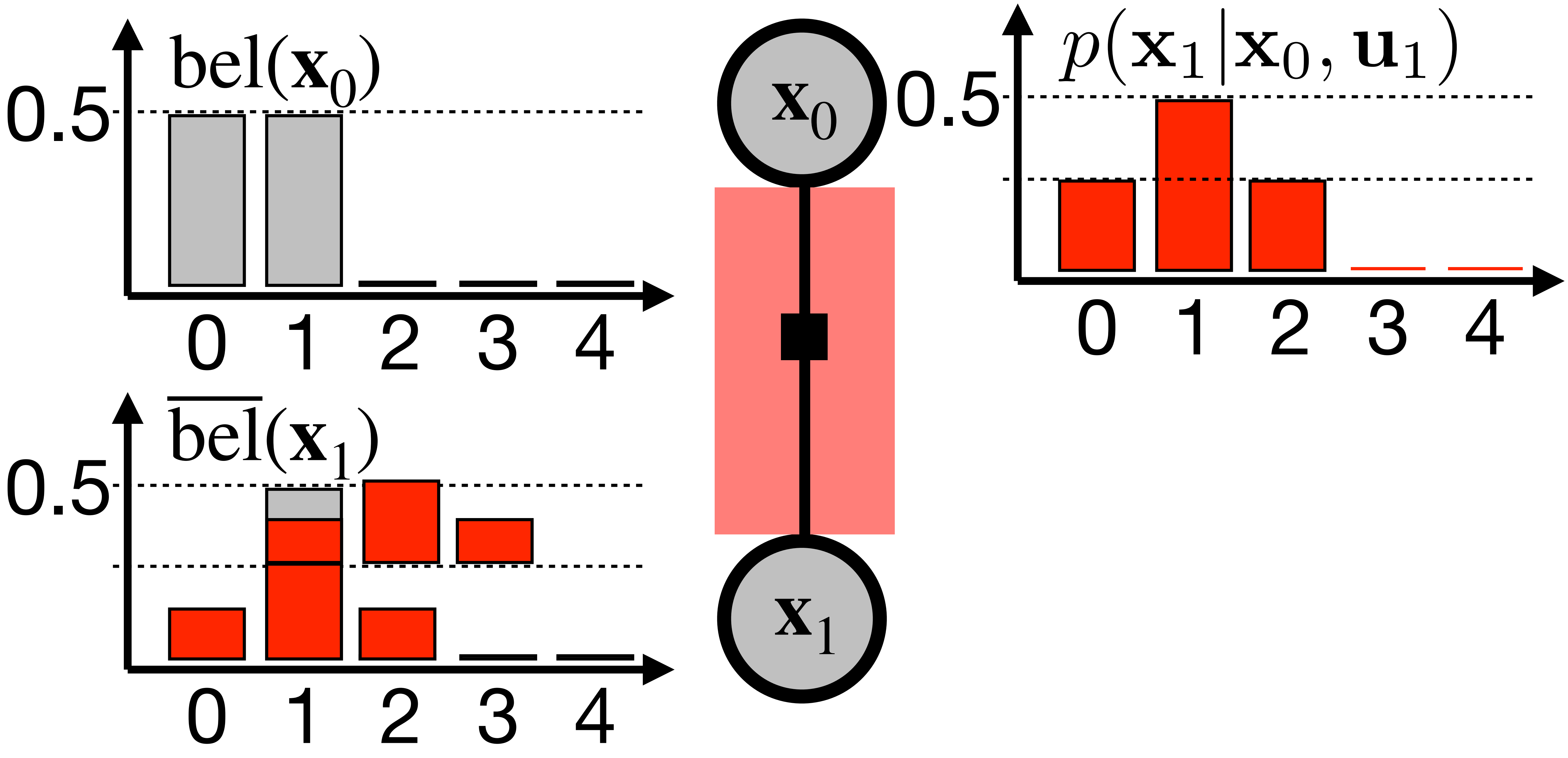
Bayes filter

Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$

Prediction step (action $\mathbf{u}_t$ performed):

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_t) \mathrm{bel}(\mathbf{x}_{t-1}) d\mathbf{x}_{t-1}$$

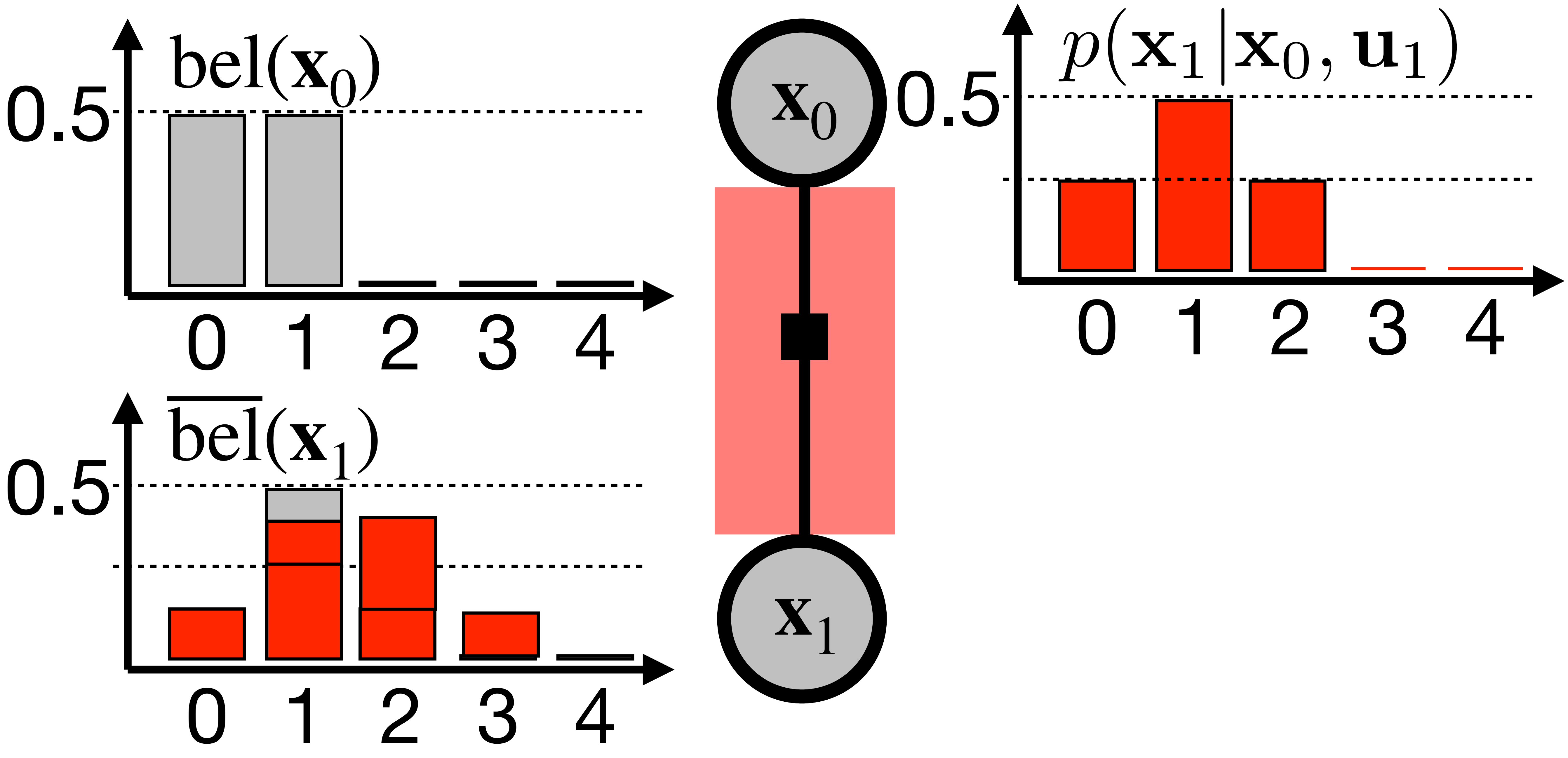


Bayes filter

Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$

Prediction step (action $\mathbf{u}_t$ performed):

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_t) \mathrm{bel}(\mathbf{x}_{t-1}) d\mathbf{x}_{t-1}$$

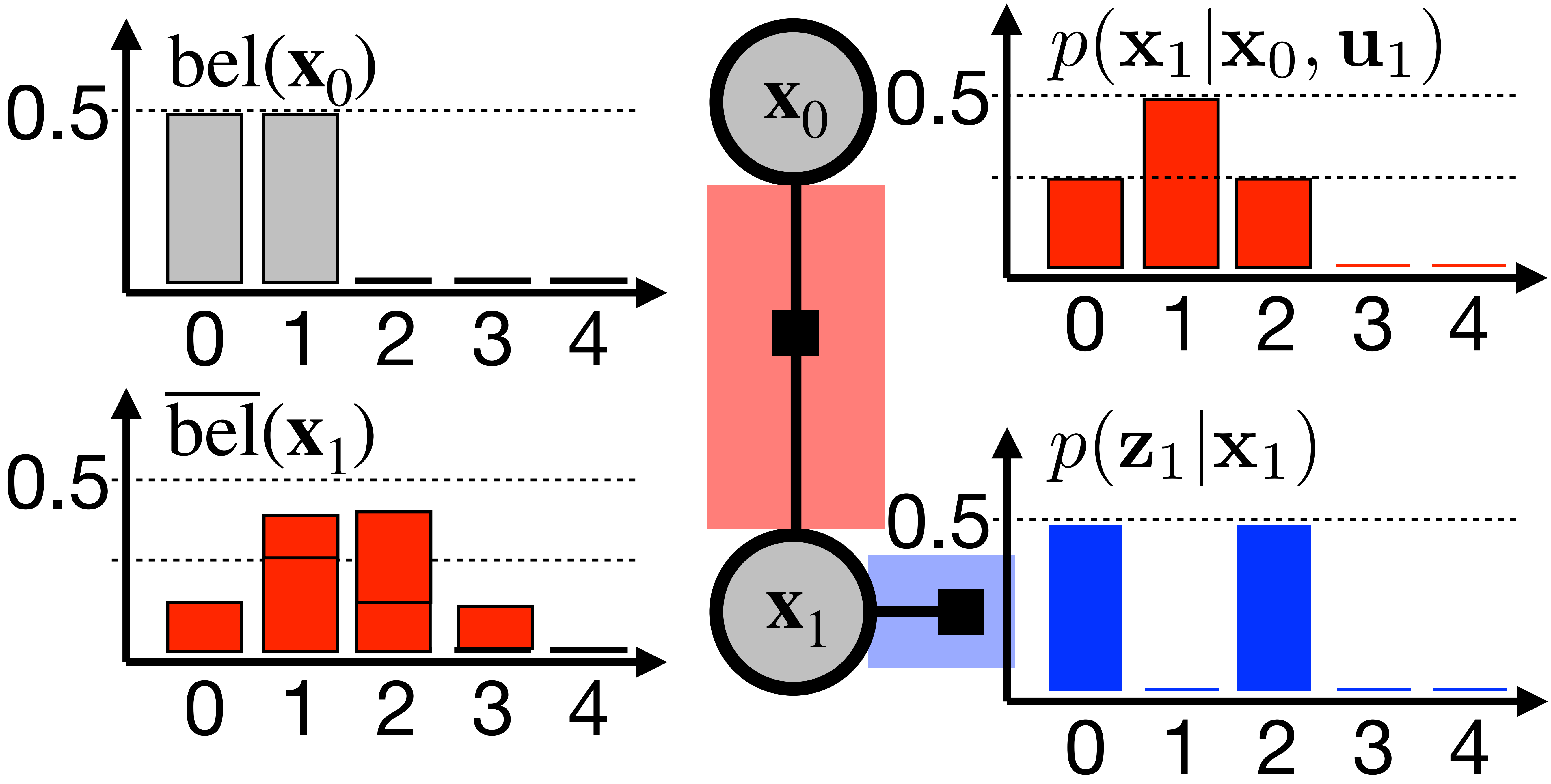


Bayes filter

Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$

Prediction step (action $\mathbf{u}_t$ performed):

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t)\mathrm{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}$$



How do you update probability distribution of x1 after the blue measurement?

Bayes filter

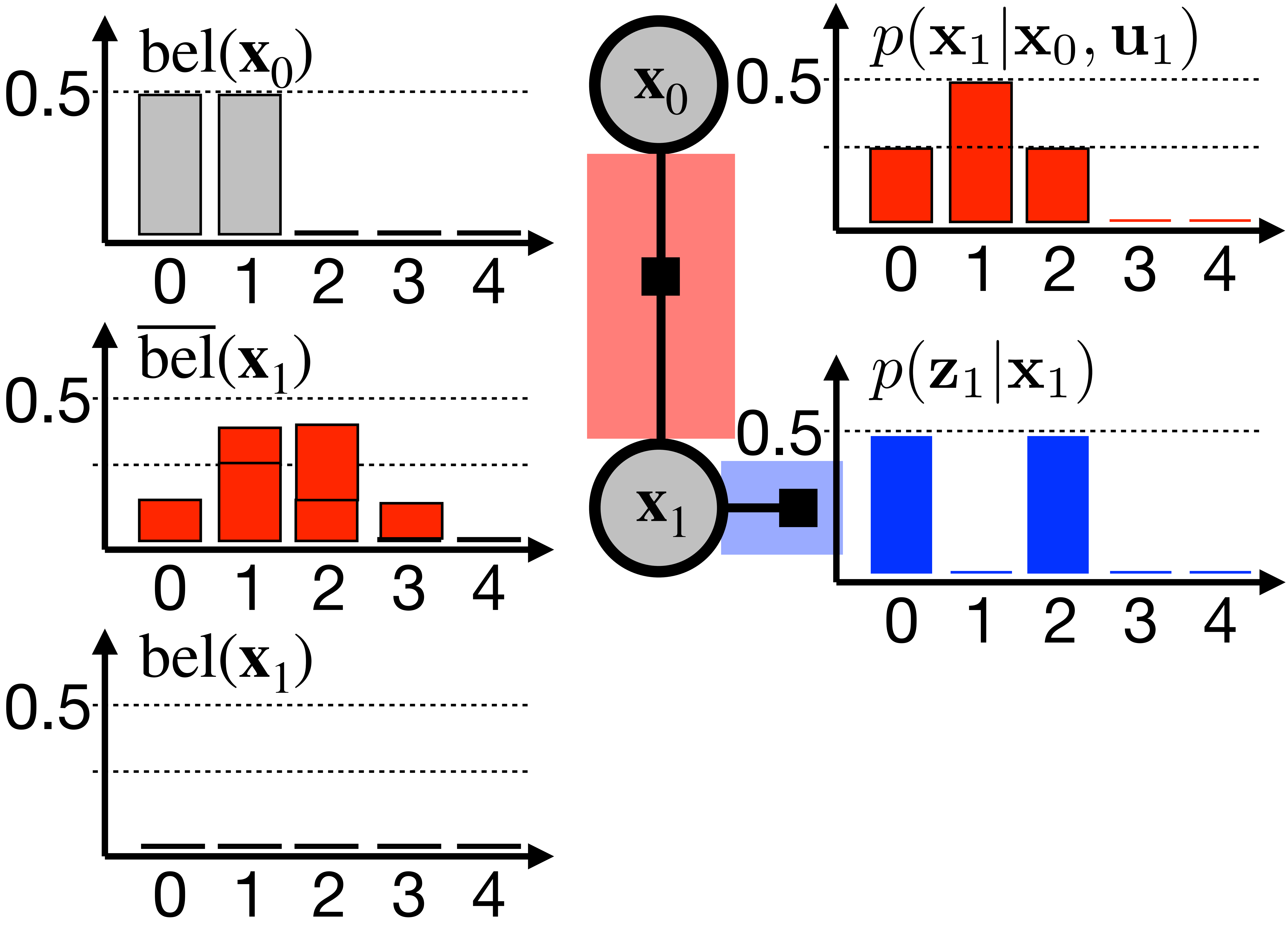
Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$

Prediction step (action $\mathbf{u}_t$ performed):

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t)\mathrm{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}$$

Measurement update (new $\mathbf{z}_t$ received):

$$\mathrm{bel}(\mathbf{x}_t) = \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t)\overline{\mathrm{bel}}(\mathbf{x}_t)$$



Bayes filter

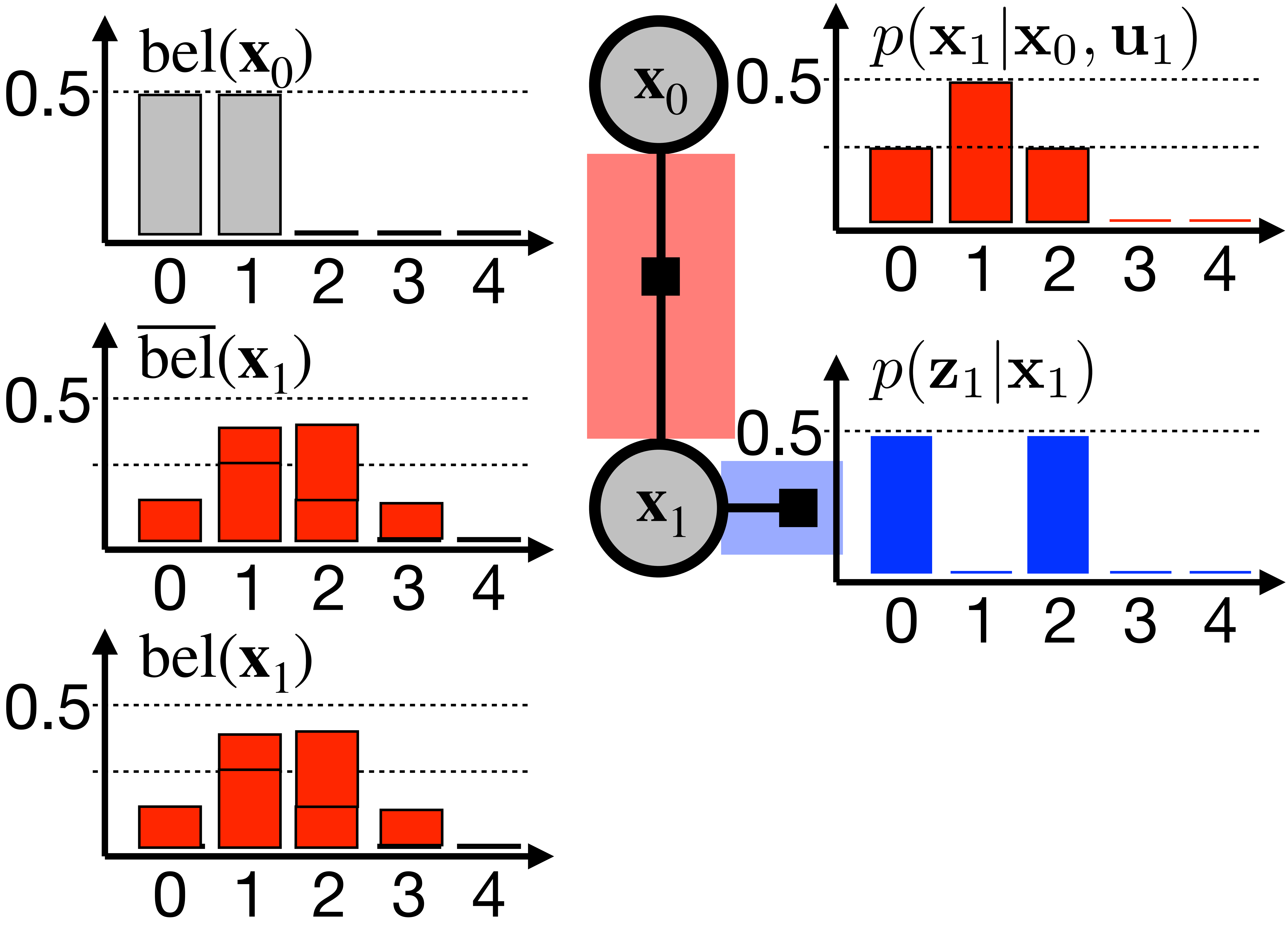
Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$

Prediction step (action $\mathbf{u}_t$ performed):

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t)\mathrm{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}$$

Measurement update (new $\mathbf{z}_t$ received):

$$\mathrm{bel}(\mathbf{x}_t) = \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t)\overline{\mathrm{bel}}(\mathbf{x}_t)$$



Bayes filter

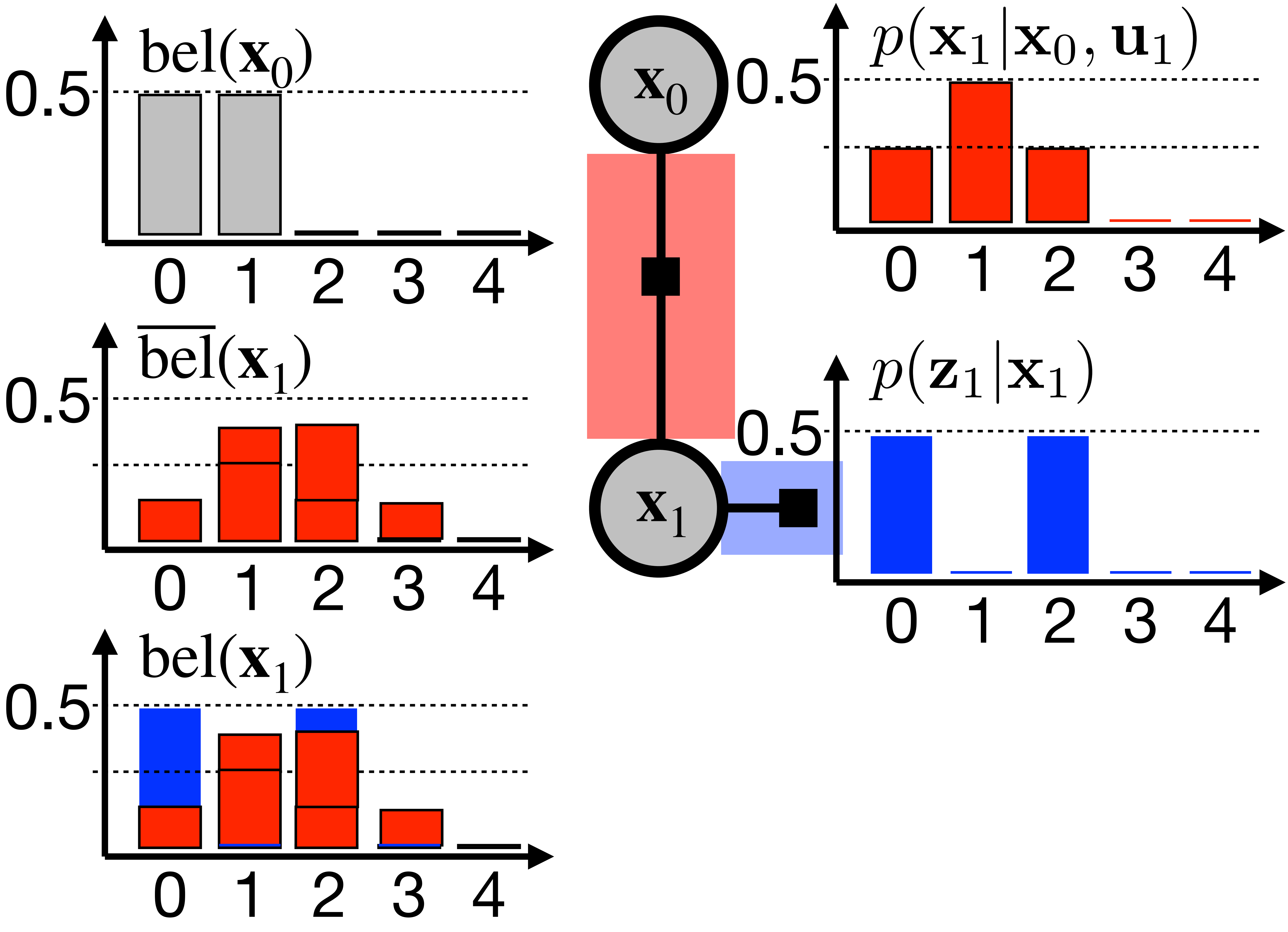
Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$

Prediction step (action $\mathbf{u}_t$ performed):

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t)\mathrm{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}$$

Measurement update (new $\mathbf{z}_t$ received):

$$\mathrm{bel}(\mathbf{x}_t) = \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t)\overline{\mathrm{bel}}(\mathbf{x}_t)$$



Bayes filter

Initialization: $\text{bel}(\mathbf{x}_0)$, $t = 1$

Prediction step (action $\mathbf{u}_t$ performed):

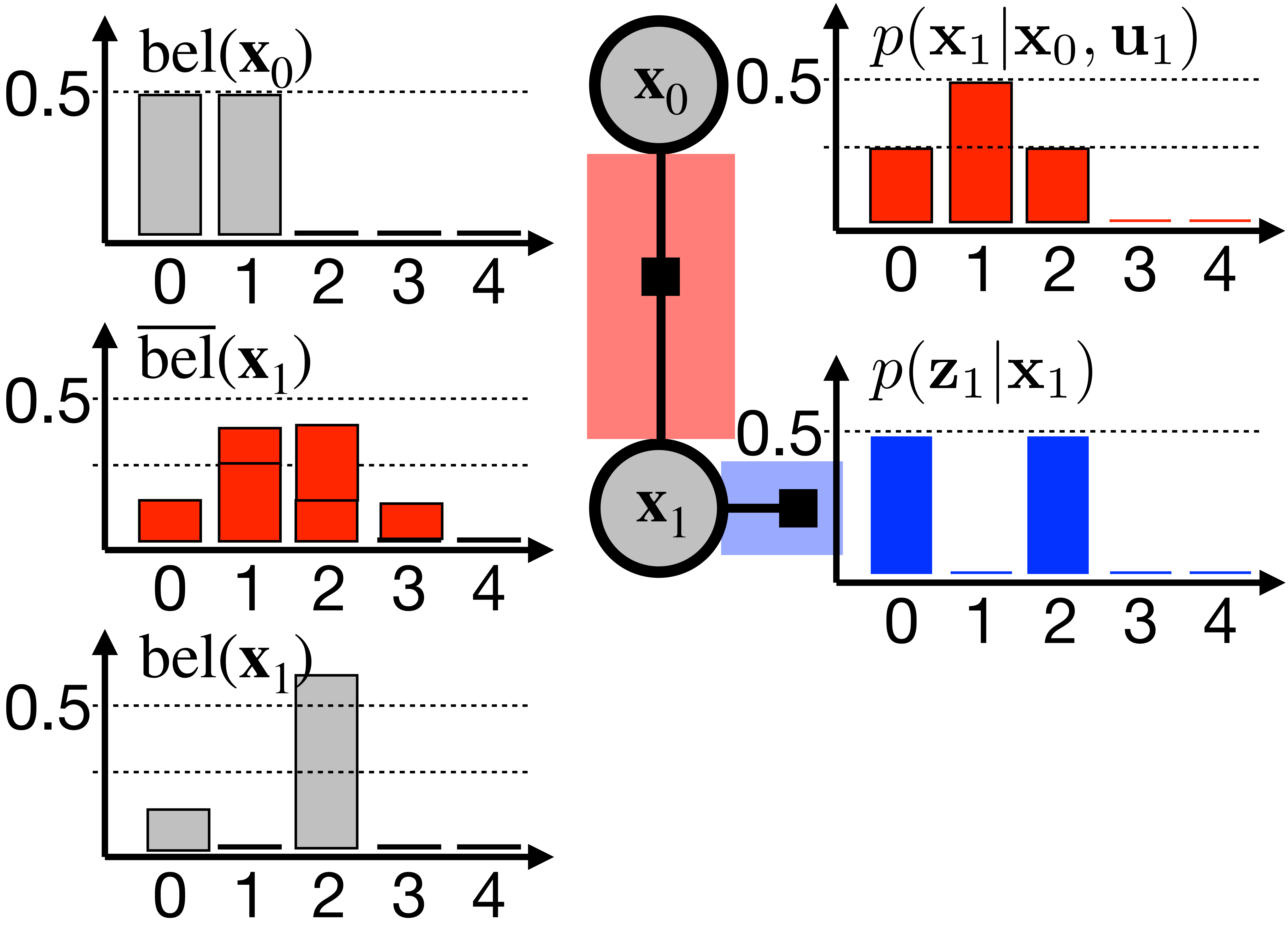
$$\overline{\text{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t)\text{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}$$

Measurement update (new $\mathbf{z}_t$ received):

$$\text{bel}(\mathbf{x}_t) = \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t)\overline{\text{bel}}(\mathbf{x}_t)$$

Repeat forever

$t = t + 1$



$\overline{\text{bel}}(\mathbf{x}_t) = p(\mathbf{x}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$ … prior belief

(prob. distr. of current state **without** considering the current measurement $\mathbf{z}_t$)

Bayes filter

Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$

Prediction step (action $\mathbf{u}_t$ performed):

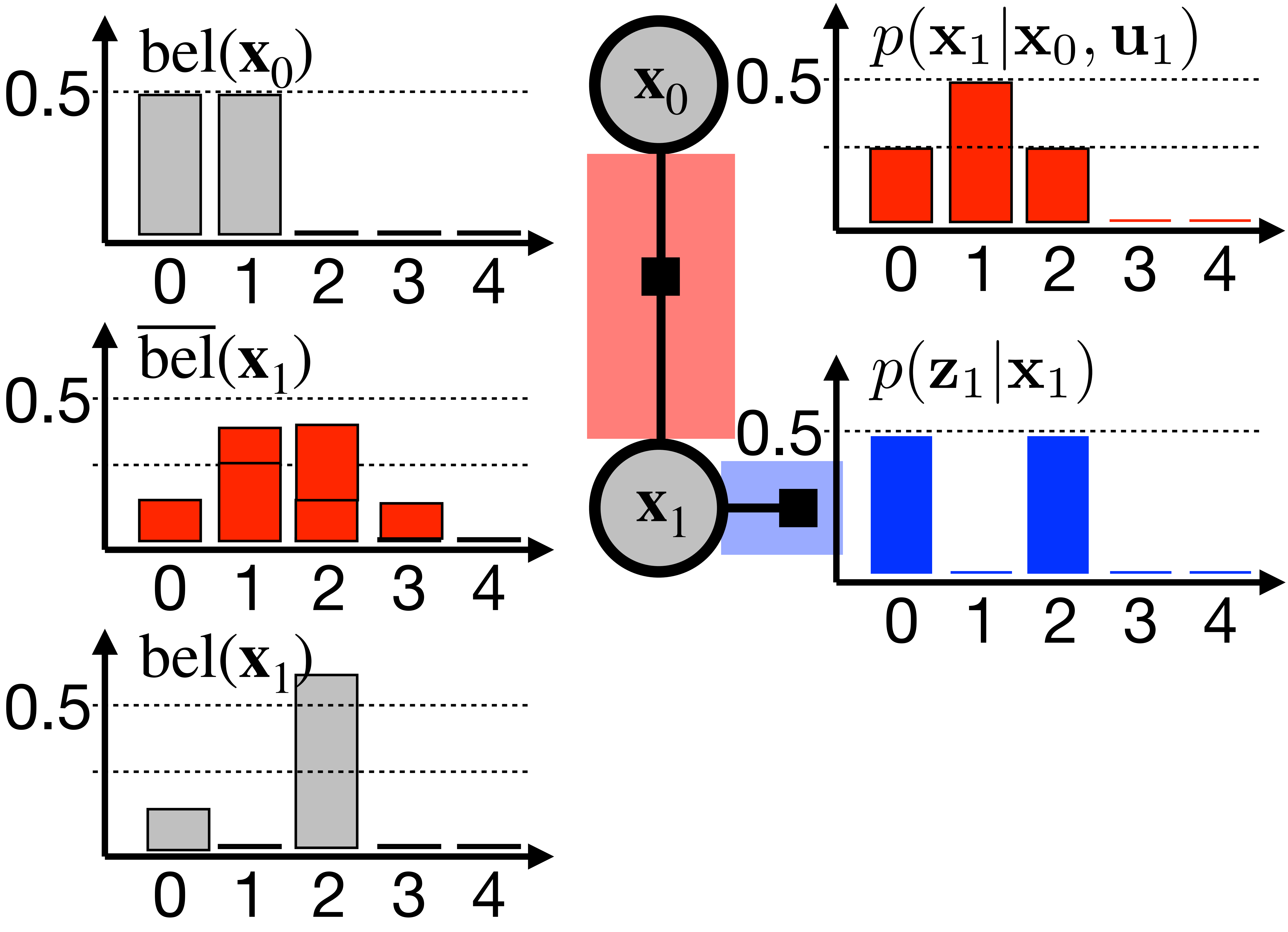
$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t)\mathrm{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}$$

Measurement update (new $\mathbf{z}_t$ received):

$$\mathrm{bel}(\mathbf{x}_t) = \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t)\overline{\mathrm{bel}}(\mathbf{x}_t)$$

Repeat forever

$t = t + 1$



$\overline{\mathrm{bel}}(\mathbf{x}_t) = p(\mathbf{x}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$ … prior belief (prob. distr. of current state **without** considering the current measurement $\mathbf{z}_t$)

$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{x}_t|\mathbf{z}_{1:t}, \mathbf{u}_{1:t})$ … posterior belief (prob. distr. of current state **with** considering the current measurement $\mathbf{z}_t$)

Bayes filter - derivation

<u>Consequences:</u>

measurement probability: $p(\mathbf{z}_t|\mathbf{x}_t) = p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{x}_{0:t-1}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$

state-transition probability: $p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) = p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t, \mathbf{x}_{0:t-2}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t-1})$

$$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{x}_t|\mathbf{z}_{1:t}, \mathbf{u}_{1:t}) = p(\mathbf{x}_t|\mathbf{z}_t, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$$

$$\overset{\text{BR}}{=} \frac{p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) \cdot p(\mathbf{x}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})}{p(\mathbf{z}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})}$$

Bayes filter - derivation

Consequences:

measurement probability: $p(\mathbf{z}_t|\mathbf{x}_t) = p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{x}_{0:t-1}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$

state-transition probability: $p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) = p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t, \mathbf{x}_{0:t-2}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t-1})$

$$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{x}_t|\mathbf{z}_{1:t}, \mathbf{u}_{1:t}) = p(\mathbf{x}_t|\mathbf{z}_t, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$$

$$\overset{\text{BR}}{=} \frac{p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) \cdot p(\mathbf{x}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})}{p(\mathbf{z}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})} \overset{\text{CI}}{=} \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot p(\mathbf{x}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$$

Bayes filter - derivation

Consequences:

measurement probability: $p(\mathbf{z}_t|\mathbf{x}_t) = p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{x}_{0:t-1}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$

state-transition probability: $p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) = p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t, \mathbf{x}_{0:t-2}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t-1})$

$$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{x}_t|\mathbf{z}_{1:t}, \mathbf{u}_{1:t}) = p(\mathbf{x}_t|\mathbf{z}_t, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$$

$$\overset{\text{BR}}{=} \frac{p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) \cdot p(\mathbf{x}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})}{p(\mathbf{z}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})} \overset{\text{CI}}{=} \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot p(\mathbf{x}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$$

$$\overset{\text{LTP}}{=} \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) \cdot p(\mathbf{x}_{t-1}|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) d\mathbf{x}_{t-1}$$

Bayes filter - derivation

Consequences:

measurement probability: $p(\mathbf{z}_t|\mathbf{x}_t) = p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{x}_{0:t-1}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$

state-transition probability: $p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) = p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t, \mathbf{x}_{0:t-2}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t-1})$

$$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{x}_t|\mathbf{z}_{1:t}, \mathbf{u}_{1:t}) = p(\mathbf{x}_t|\mathbf{z}_t, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$$

$$\overset{\text{BR}}{=} \frac{p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) \cdot p(\mathbf{x}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})}{p(\mathbf{z}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})} \overset{\text{CI}}{=} \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot p(\mathbf{x}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$$

$$\overset{\text{LTP}}{=} \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) \cdot p(\mathbf{x}_{t-1}|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) d\mathbf{x}_{t-1}$$

$$\overset{\text{CI}}{=} \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) \cdot p(\mathbf{x}_{t-1}|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) d\mathbf{x}_{t-1}$$

Bayes filter - derivation

Consequences:

measurement probability: $p(\mathbf{z}_t|\mathbf{x}_t) = p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{x}_{0:t-1}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$

state-transition probability: $p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) = p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t, \mathbf{x}_{0:t-2}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t-1})$

$$\text{bel}(\mathbf{x}_t) = p(\mathbf{x}_t|\mathbf{z}_{1:t}, \mathbf{u}_{1:t}) = p(\mathbf{x}_t|\mathbf{z}_t, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$$

$$\overset{\text{BR}}{=} \frac{p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) \cdot p(\mathbf{x}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})}{p(\mathbf{z}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})} \overset{\text{CI}}{=} \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot p(\mathbf{x}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$$

$$\overset{\text{LTP}}{=} \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) \cdot p(\mathbf{x}_{t-1}|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) d\mathbf{x}_{t-1}$$

$$\overset{\text{CI}}{=} \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) \cdot p(\mathbf{x}_{t-1}|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) d\mathbf{x}_{t-1}$$

$$= \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \text{bel}(\mathbf{x}_{t-1}) d\mathbf{x}_{t-1}$$

Bayes filter - derivation

Consequences:

measurement probability: $p(\mathbf{z}_t|\mathbf{x}_t) = p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{x}_{0:t-1}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$

state-transition probability: $p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) = p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t, \mathbf{x}_{0:t-2}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t-1})$

$$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{x}_t|\mathbf{z}_{1:t}, \mathbf{u}_{1:t}) = p(\mathbf{x}_t|\mathbf{z}_t, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$$

$$\overset{\text{BR}}{=} \frac{p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) \cdot p(\mathbf{x}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})}{p(\mathbf{z}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})} \overset{\text{CI}}{=} \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot p(\mathbf{x}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$$

$$\overset{\text{LTP}}{=} \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) \cdot p(\mathbf{x}_{t-1}|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) d\mathbf{x}_{t-1}$$

$$\overset{\text{CI}}{=} \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) \cdot p(\mathbf{x}_{t-1}|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) d\mathbf{x}_{t-1}$$

$$= \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1}) d\mathbf{x}_{t-1}$$

Bayes filter - derivation

Consequences:

measurement probability: $p(\mathbf{z}_t|\mathbf{x}_t) = p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{x}_{0:t-1}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$

state-transition probability: $p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) = p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t, \mathbf{x}_{0:t-2}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t-1})$

$$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{x}_t|\mathbf{z}_{1:t}, \mathbf{u}_{1:t}) = p(\mathbf{x}_t|\mathbf{z}_t, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$$

$$\overset{\text{BR}}{=} \frac{p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) \cdot p(\mathbf{x}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})}{p(\mathbf{z}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})} \overset{\text{CI}}{=} \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot p(\mathbf{x}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$$

$$\overset{\text{LTP}}{=} \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) \cdot p(\mathbf{x}_{t-1}|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) d\mathbf{x}_{t-1}$$

$$\overset{\text{CI}}{=} \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) \cdot p(\mathbf{x}_{t-1}|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) d\mathbf{x}_{t-1}$$

$$= \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot \overbrace{\int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1}) d\mathbf{x}_{t-1}}^{\overline{\mathrm{bel}}(\mathbf{x}_t)}$$

$$\text{bel}(\mathbf{x}_t) = \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot \underbrace{\int p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) \cdot \text{bel}(\mathbf{x}_{t-1}) d\mathbf{x}_{t-1}}_{\overline{\text{bel}}(\mathbf{x}_t)}$$

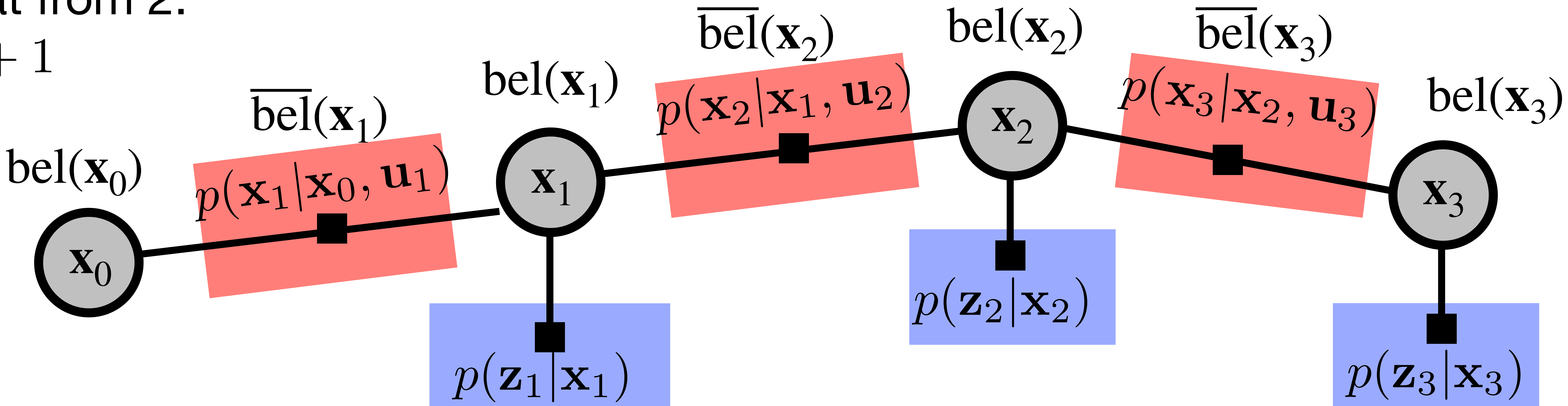
1. Initialization: $\text{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

$$\overline{\text{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t)\text{bel}(\mathbf{x}_{t-1}) d\mathbf{x}_{t-1}$$

3. Measurement update (new $\mathbf{z}_t$ received):

$$\text{bel}(\mathbf{x}_t) = \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t)\overline{\text{bel}}(\mathbf{x}_t)$$

4. Repeat from 2:
   $t = t + 1$



$$\text{bel}(\mathbf{x}_t) = \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot \underbrace{\int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \text{bel}(\mathbf{x}_{t-1}) d\mathbf{x}_{t-1}}_{\overline{\text{bel}}(\mathbf{x}_t)}$$

1. Initialization: $\text{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

$$\overline{\text{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t)\text{bel}(\mathbf{x}_{t-1}) d\mathbf{x}_{t-1}$$

3. Measurement update (new $\mathbf{z}_t$ received):

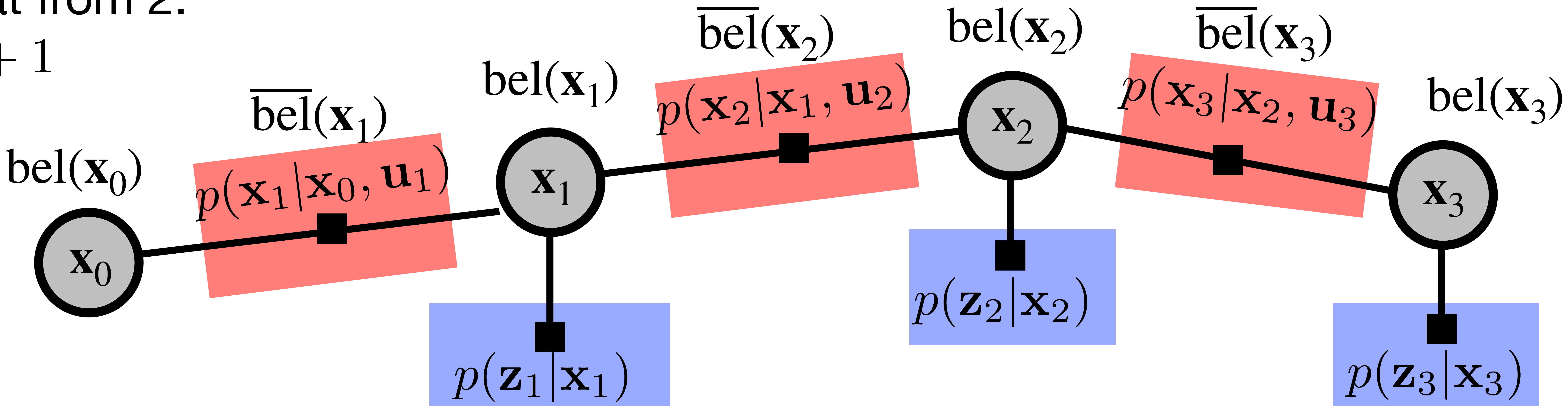
$$\text{bel}(\mathbf{x}_t) = \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t)\overline{\text{bel}}(\mathbf{x}_t)$$

4. Repeat from 2:
$t = t + 1$

Is there any obvious limitation of dicrete prob. distribution?

Discrete probability distribution will suffer from curse of dimensionality

=> Let's return to Gaussians in continuous space



System model

Linear system:

$$\mathbf{x}_t = \mathbf{A}_t \mathbf{x}_{t-1} + \mathbf{B}_t \mathbf{u}_t$$

$$\mathbf{z}_t = \mathbf{C}_t \mathbf{x}_t$$

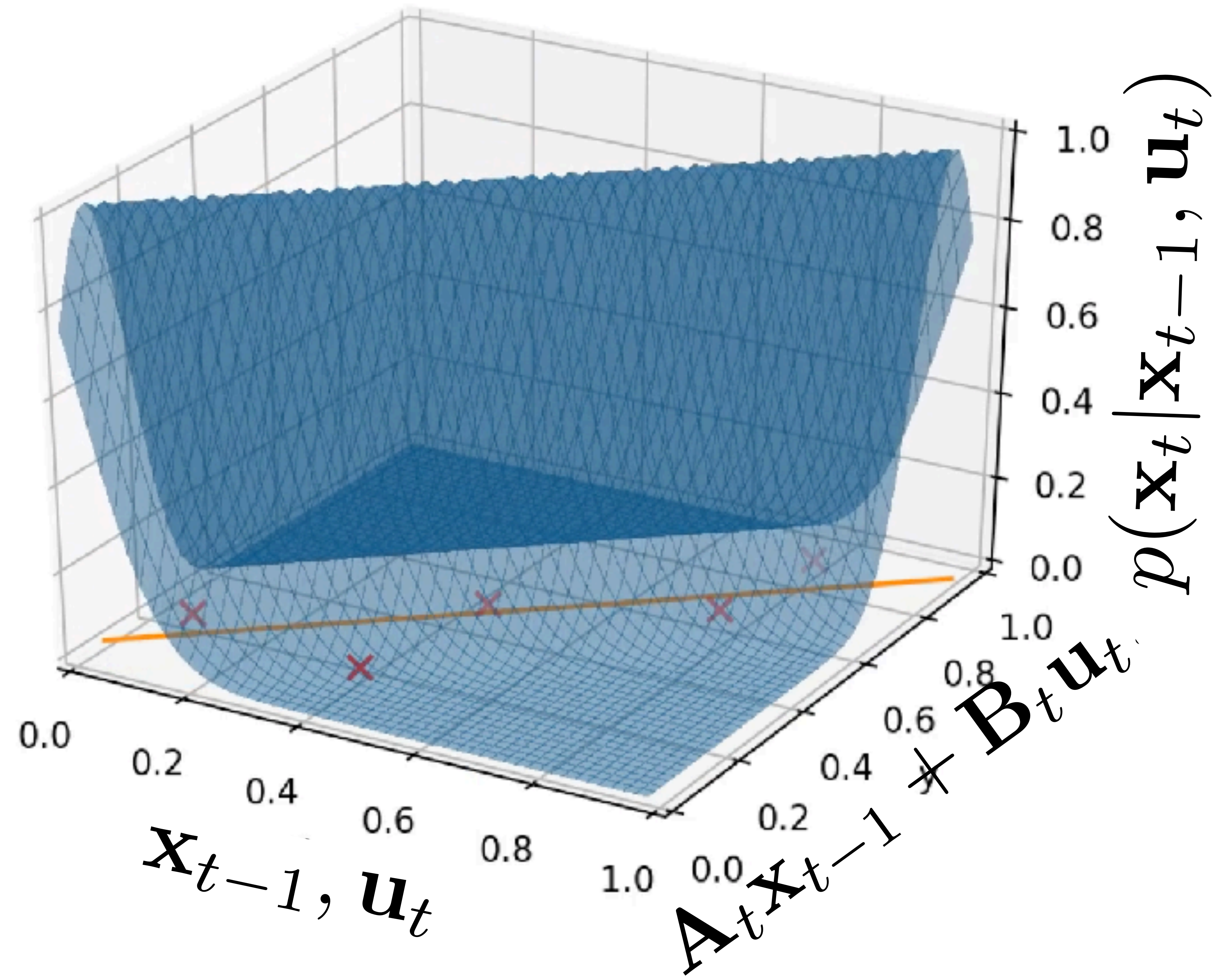
Let's add Gaussian noise

System model

Linear system with Gaussian noise:

$$p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) = \mathcal{N}_{\mathbf{x}_t}(\mathbf{A}_t\mathbf{x}_{t-1} + \mathbf{B}_t\mathbf{u}_t,\ \mathbf{R}_t)$$

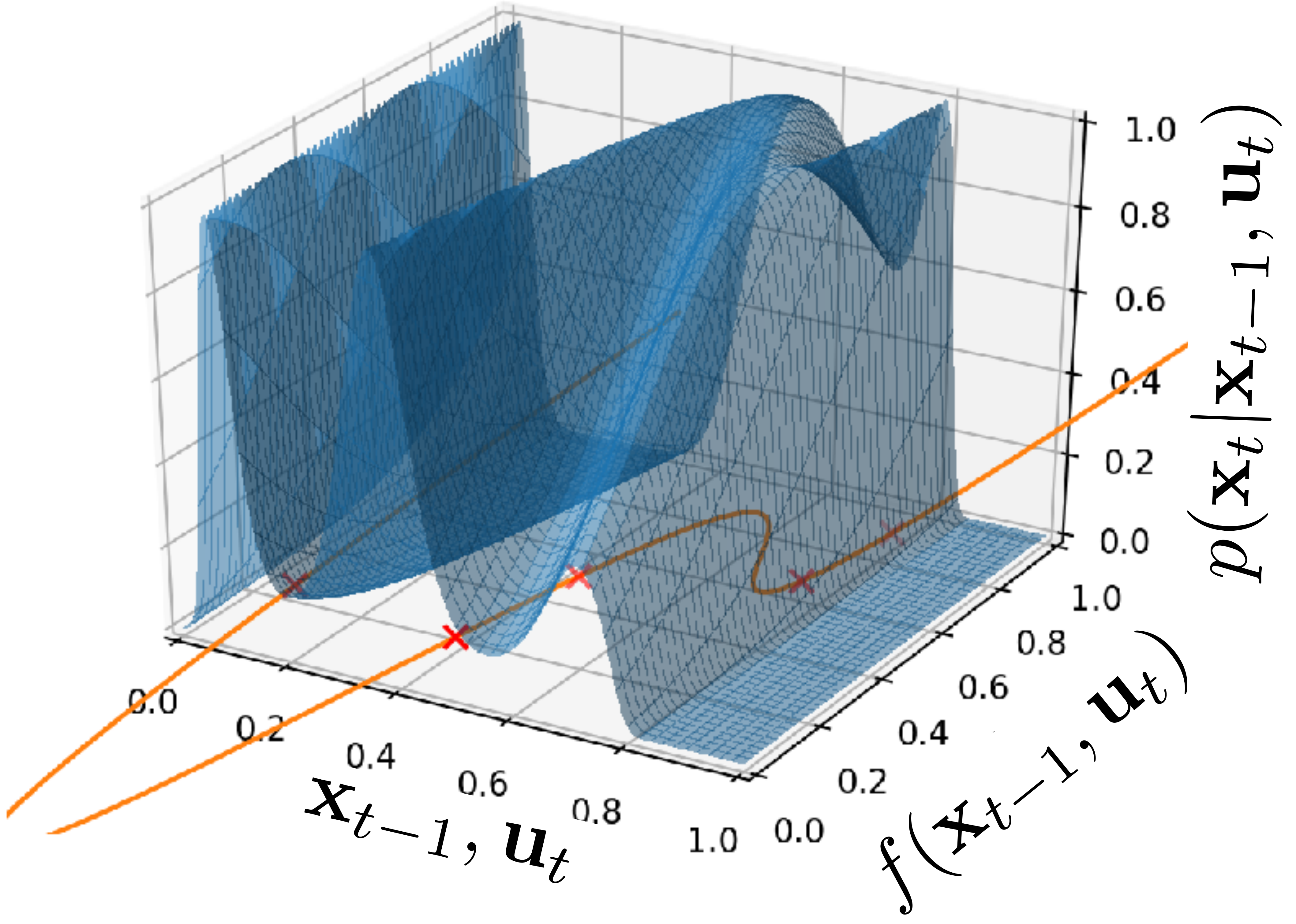
$$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}_{\mathbf{z}_t}(\mathbf{C}_t\mathbf{x}_t,\ \mathbf{Q}_t)$$



Non-linear system with Gaussian noise:

$$p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) = \mathcal{N}_{\mathbf{x}_t}(f(\mathbf{x}_{t-1},\mathbf{u}_t),\ \mathbf{R}_t)$$

$$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}_{\mathbf{z}_t}(h(\mathbf{x}_t),\ \mathbf{Q}_t)$$



Linear system with Gaussian noise: $p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) = \mathcal{N}_{\mathbf{x}_t}(\mathbf{A}_t\mathbf{x}_{t-1} + \mathbf{B}_t\mathbf{u}_t,\ \mathbf{R}_t)$

$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}_{\mathbf{z}_t}(\mathbf{C}_t\mathbf{x}_t,\ \mathbf{Q}_t)$

Gaussian

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t)\mathrm{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}$$

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$$
$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t\boldsymbol{\mu}_{t-1} + \mathbf{B}_t\mathbf{u}_t$$
$$\overline{\boldsymbol{\Sigma}}_t = \mathbf{A}_t\boldsymbol{\Sigma}_{t-1}\mathbf{A}_t^\top + \mathbf{R}_t$$

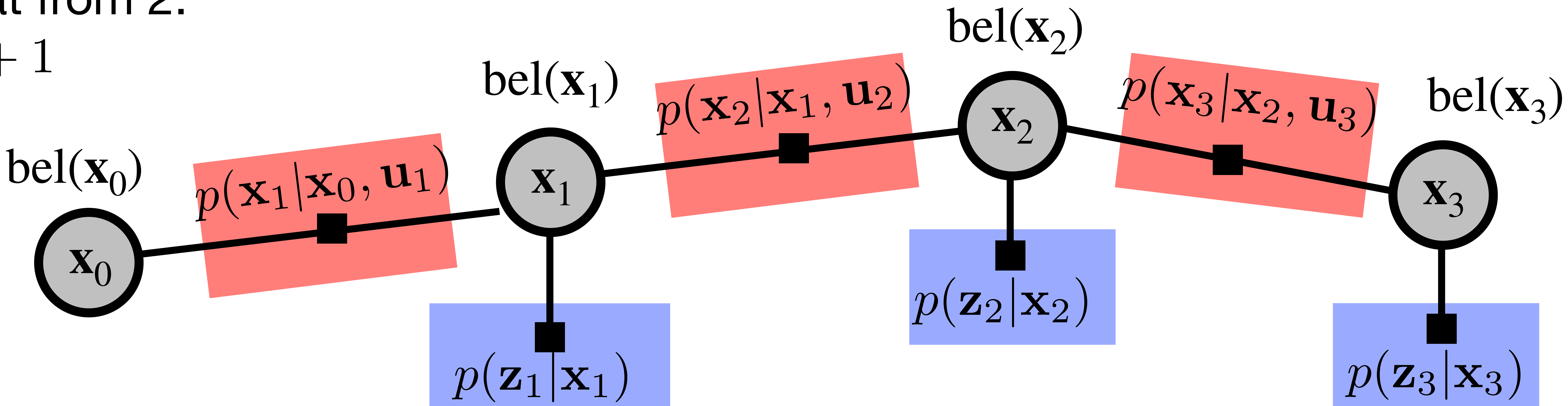
3. Measurement update (new $\mathbf{z}_t$ received):

$$\mathrm{bel}(\mathbf{x}_t) = \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t)\overline{\mathrm{bel}}(\mathbf{x}_t)$$

Intuition behind prediction step

4. Repeat from 2:

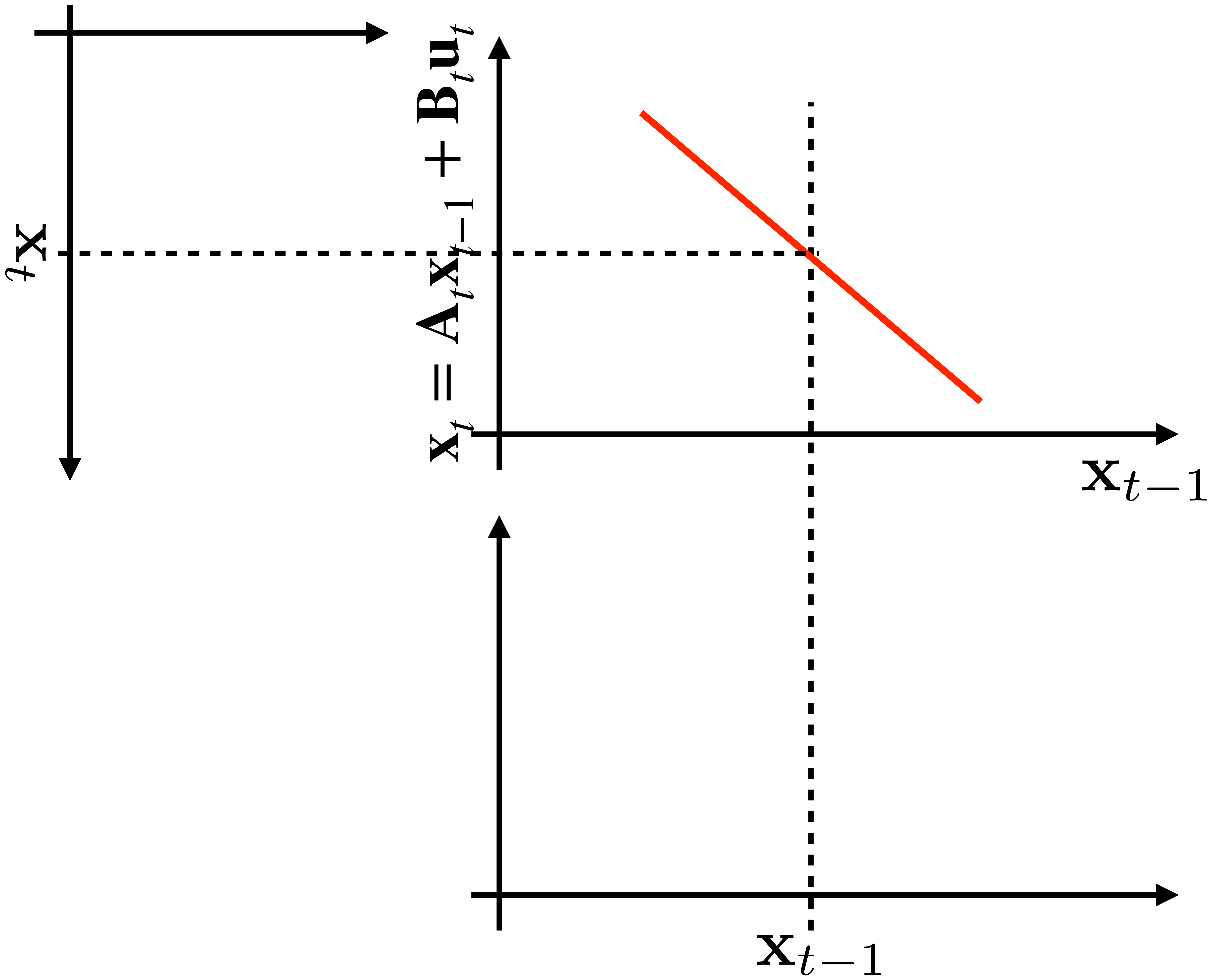
$t = t + 1$



Intuition behind prediction step

Linear system:

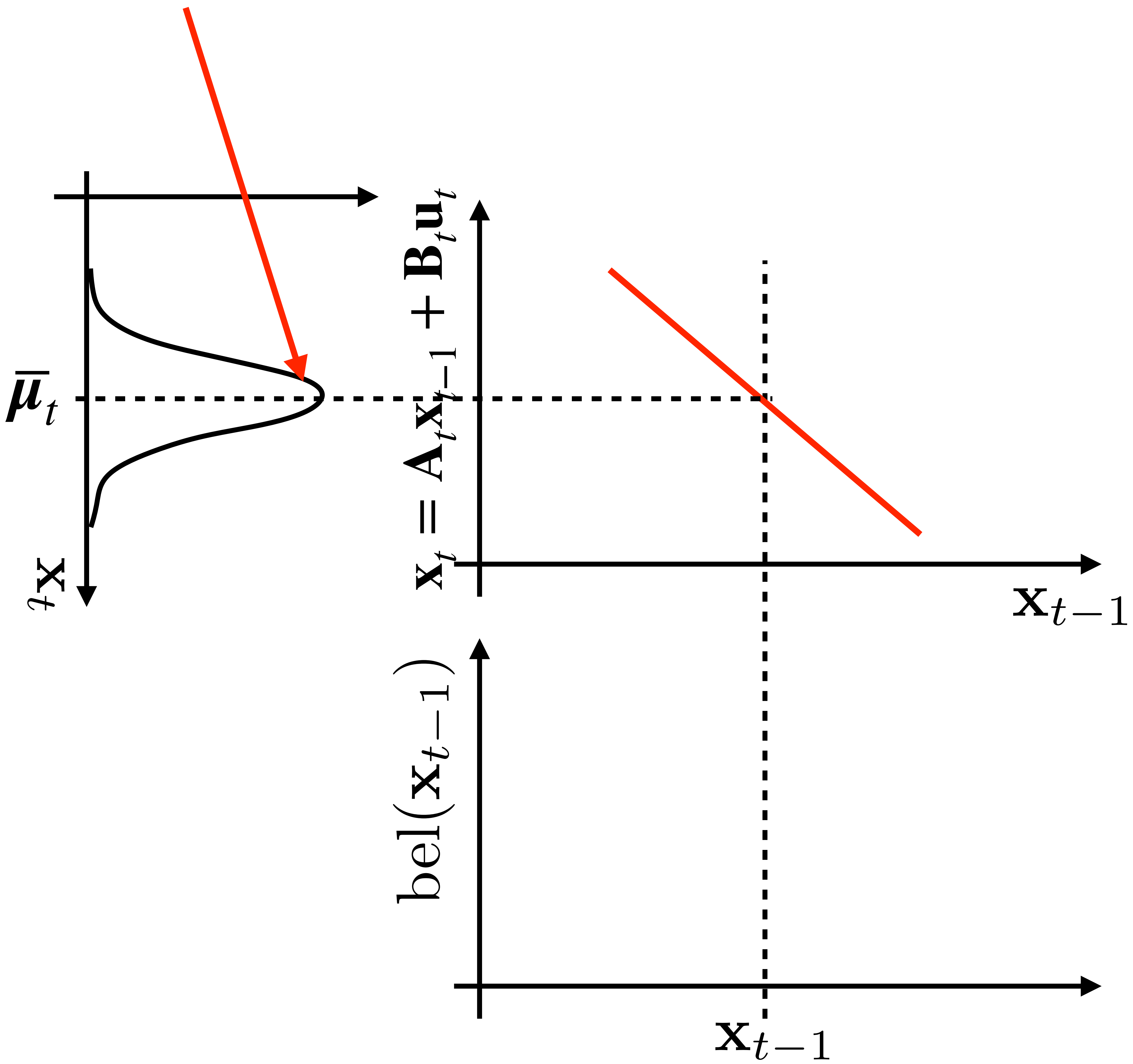
$$\mathbf{x}_t = \mathbf{A}_t \mathbf{x}_{t-1} + \mathbf{B}_t \mathbf{u}_t$$



Intuition behind prediction step

Linear system with Gaussian noise:

$$p(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{u}_t) = \mathcal{N}(\mathbf{x}_t;\ \mathbf{A}_t\mathbf{x}_{t-1} + \mathbf{B}_t\mathbf{u}_t,\ \mathbf{R}_t)$$

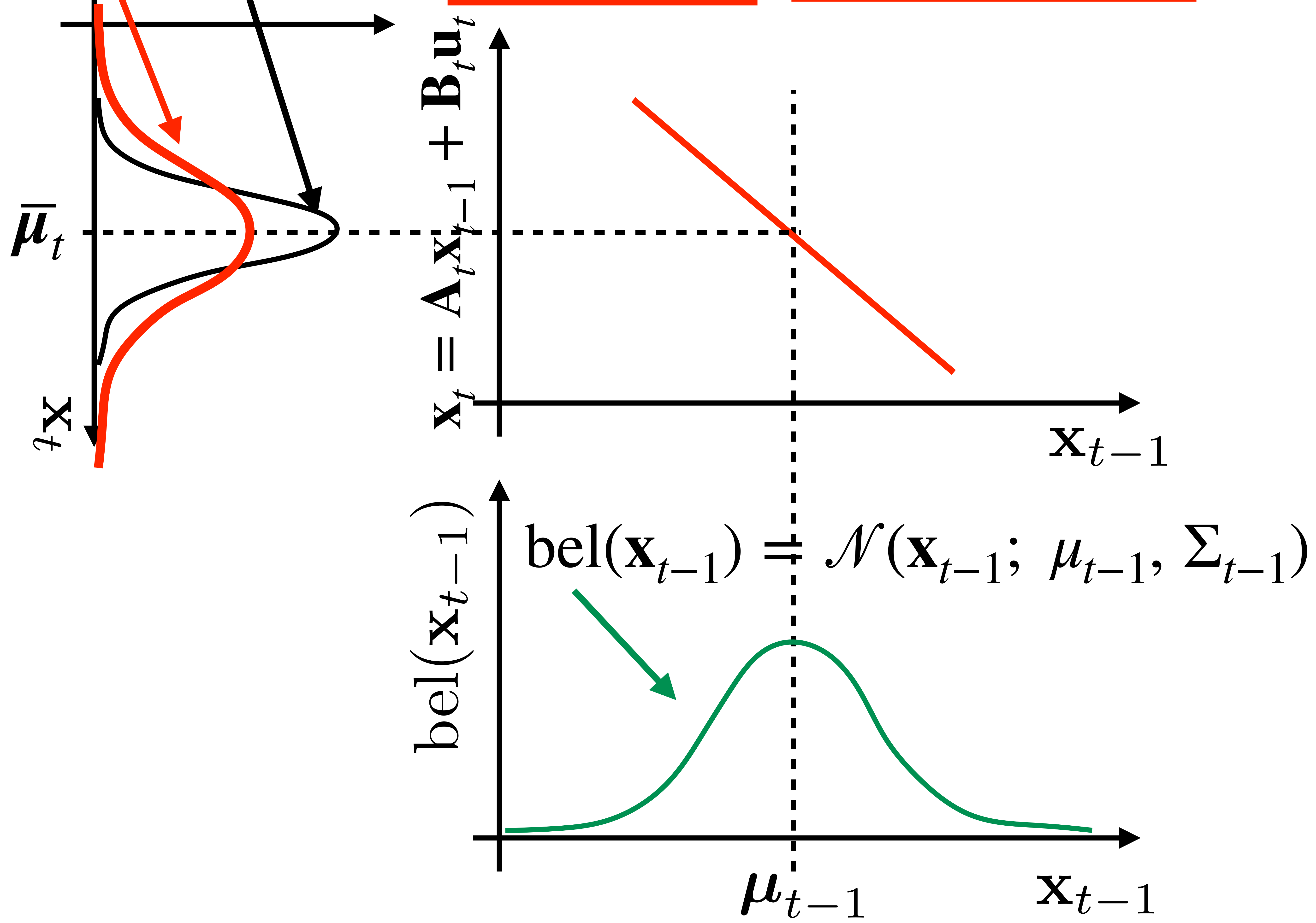


Intuition behind prediction step

Linear system with Gaussian noise:

$$p(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{u}_t) = \mathcal{N}(\mathbf{x}_t;\ \mathbf{A}_t\mathbf{x}_{t-1} + \mathbf{B}_t\mathbf{u}_t,\ \mathbf{R}_t)$$

$$\overline{\text{bel}}(\mathbf{x}_t) = \mathcal{N}(\mathbf{x}_t;\ \underbrace{\mathbf{A}\mu_{t-1} + \mathbf{B}\mathbf{u}_t}_{\bar{\boldsymbol{\mu}}_t},\ \underbrace{\mathbf{A}^\top\Sigma_{t-1}\mathbf{A}^\top + \mathbf{R}_t}_{\bar{\boldsymbol{\Sigma}}_t})$$



Expectation is linear mapping

$$\bar{\mathbf{y}} = \mathbb{E}\{\mathbf{y}\} = \mathbb{E}\{\mathbf{A}\mathbf{x} + \mathbf{b}\} = \mathbf{A}\mathbb{E}\{\mathbf{x}\} + \mathbf{b} = \mathbf{A}\bar{\mathbf{x}} + \mathbf{b}$$

Covariance is as follows:

$$\begin{aligned}
\mathbf{C}_\mathbf{y} &\triangleq \mathbb{E}\{(\mathbf{y} - \bar{\mathbf{y}})(\mathbf{y} - \bar{\mathbf{y}})^\top\} \\
&= \mathbb{E}\left\{\left[(\mathbf{A}\mathbf{x} + \mathbf{b}) - (\mathbf{A}\bar{\mathbf{x}} + \mathbf{b})\right]\left[(\mathbf{A}\mathbf{x} + \mathbf{b}) - (\mathbf{A}\bar{\mathbf{x}} + \mathbf{b})\right]^\top\right\} \\
&= \mathbb{E}\left\{\left[\mathbf{A}(\mathbf{x} - \bar{\mathbf{x}})\right]\left[\mathbf{A}(\mathbf{x} - \bar{\mathbf{x}})\right]^\top\right\} \\
&= \mathbb{E}\left\{\mathbf{A}(\mathbf{x} - \bar{\mathbf{x}})(\mathbf{x} - \bar{\mathbf{x}})^\top\mathbf{A}^\top\right\} \\
&= \mathbf{A}\mathbb{E}\left\{(\mathbf{x} - \bar{\mathbf{x}})(\mathbf{x} - \bar{\mathbf{x}})^\top\right\}\mathbf{A}^\top \\
&= \mathbf{A}\mathbf{C}_\mathbf{x}\mathbf{A}^\top
\end{aligned}$$

See section 3.2.4 in the probabilistic robotics book for the full derivation.

Linear system with Gaussian noise: $p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) = \mathcal{N}_{\mathbf{x}_t}(\mathbf{A}_t\mathbf{x}_{t-1} + \mathbf{B}_t\mathbf{u}_t,\ \mathbf{R}_t)$

$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}_{\mathbf{z}_t}(\mathbf{C}_t\mathbf{x}_t,\ \mathbf{Q}_t)$

Gaussian

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

$\overline{\mathrm{bel}}(\mathbf{x}_t) = \int$ … $)\mathrm{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}$

3. Measure… $\mathbf{z}_t$ received):

$\mathrm{bel}(\mathbf{x}_t) =$ … $\mathbf{x}_t)$

4. Repeat f…

$t = t + 1$

https://cs.wikipedia.org/wiki/Thomas_Bayes

$\overline{\mathrm{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$

$\overline{\boldsymbol{\mu}}_t$

$\overline{\boldsymbol{\Sigma}}_t$

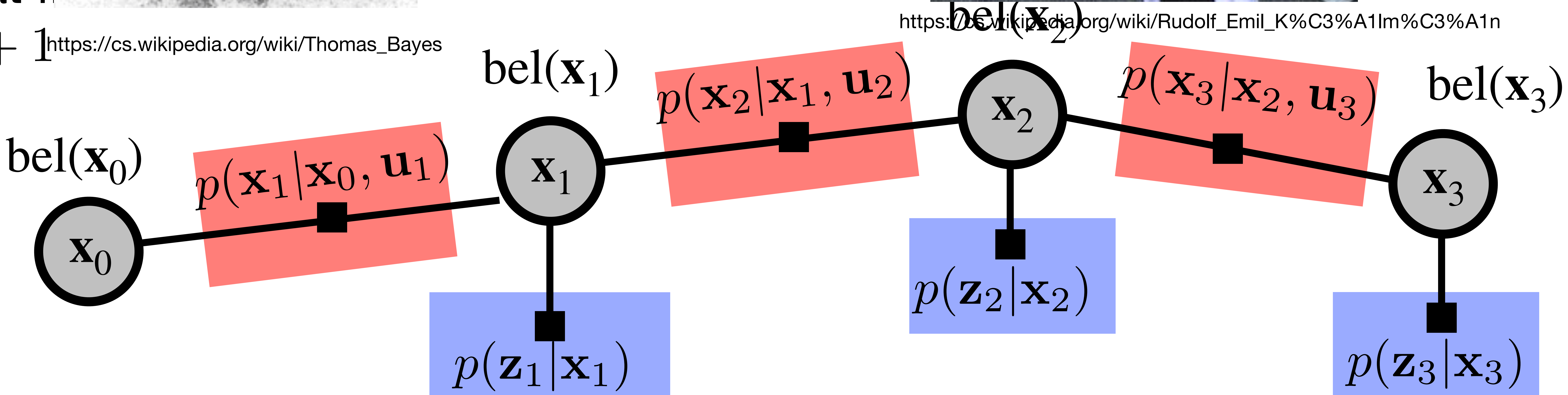
bel

$\mathbf{K}_t$ … $+ \mathbf{Q}_t)^{-1}$

$\boldsymbol{\mu}_t$ … $\overline{\boldsymbol{\mu}}_t)$

$\boldsymbol{\Sigma}_t$

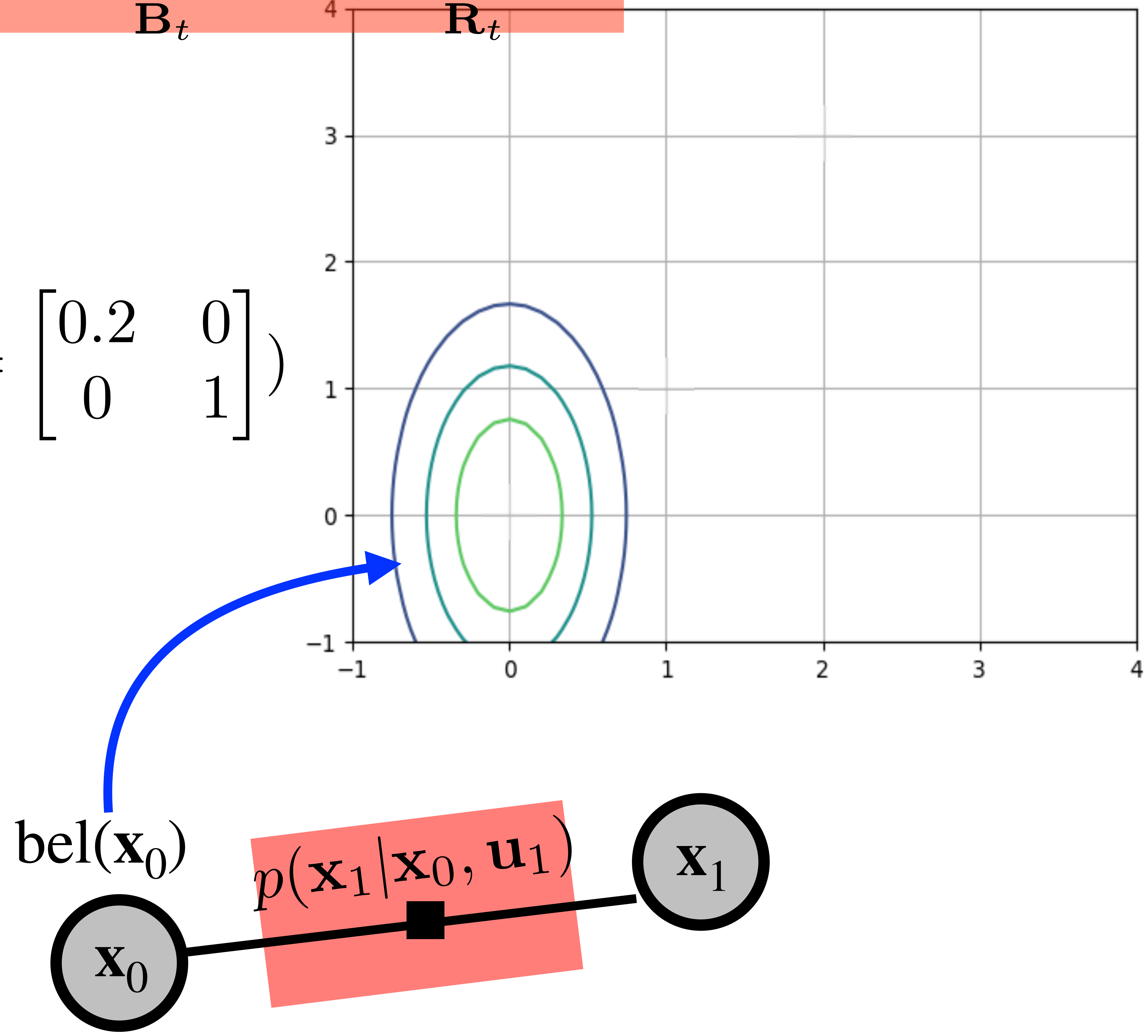
https://cs.wikipedia.org/wiki/Rudolf_Emil_K%C3%A1lm%C3%A1n



$$p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) = \mathcal{N}\Big(\mathbf{x}_t;\ \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{A}_t}\mathbf{x}_{t-1} + \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{B}_t}\mathbf{u}_t,\ \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{R}_t}\Big)$$

1. Initialization:

$$\mathrm{bel}(\mathbf{x}_0) = \mathcal{N}(\mathbf{x}_t;\ \boldsymbol{\mu}_o = \begin{bmatrix}0\\0\end{bmatrix}, \boldsymbol{\Sigma}_0 = \begin{bmatrix}0.2 & 0\\ 0 & 1\end{bmatrix})$$

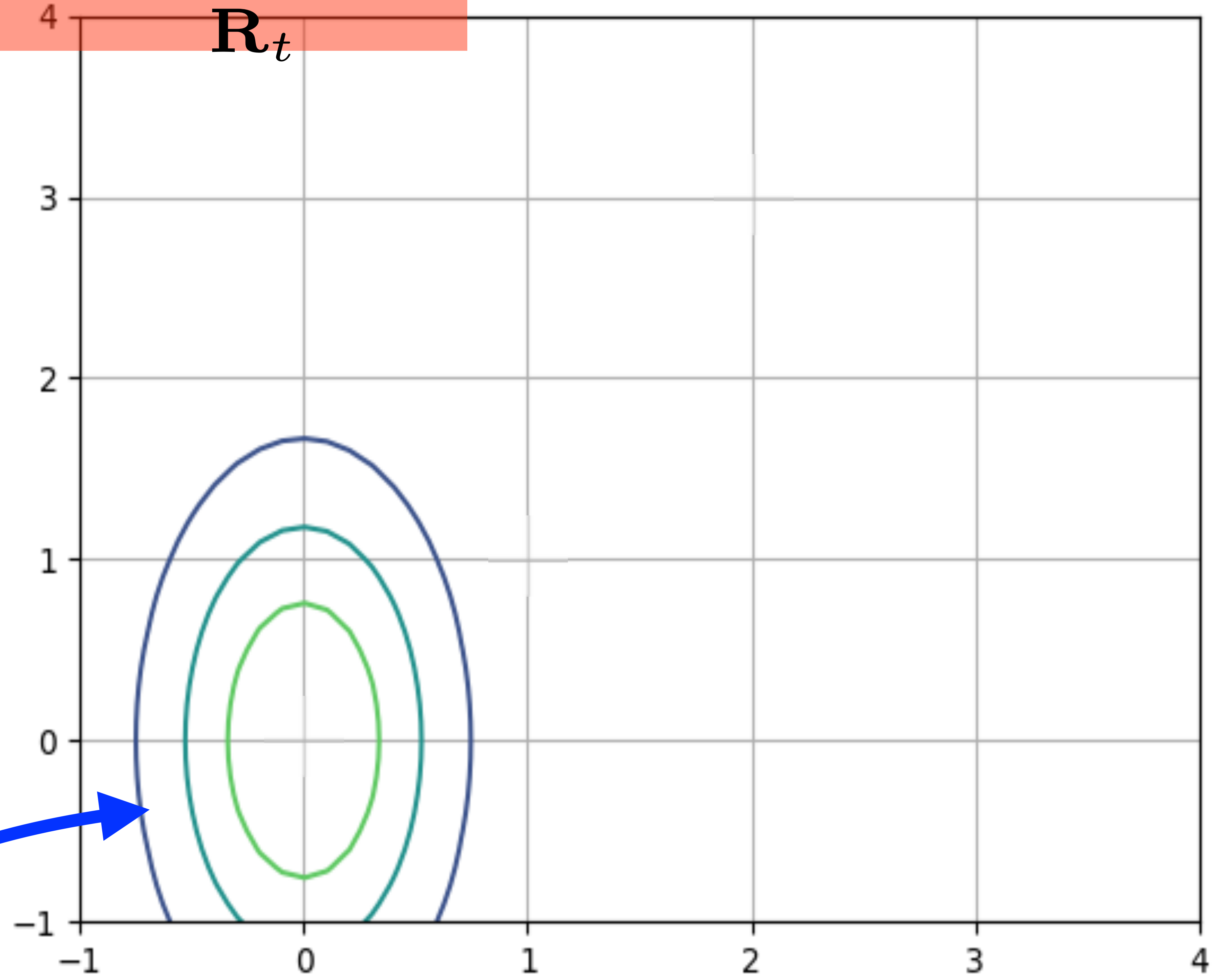


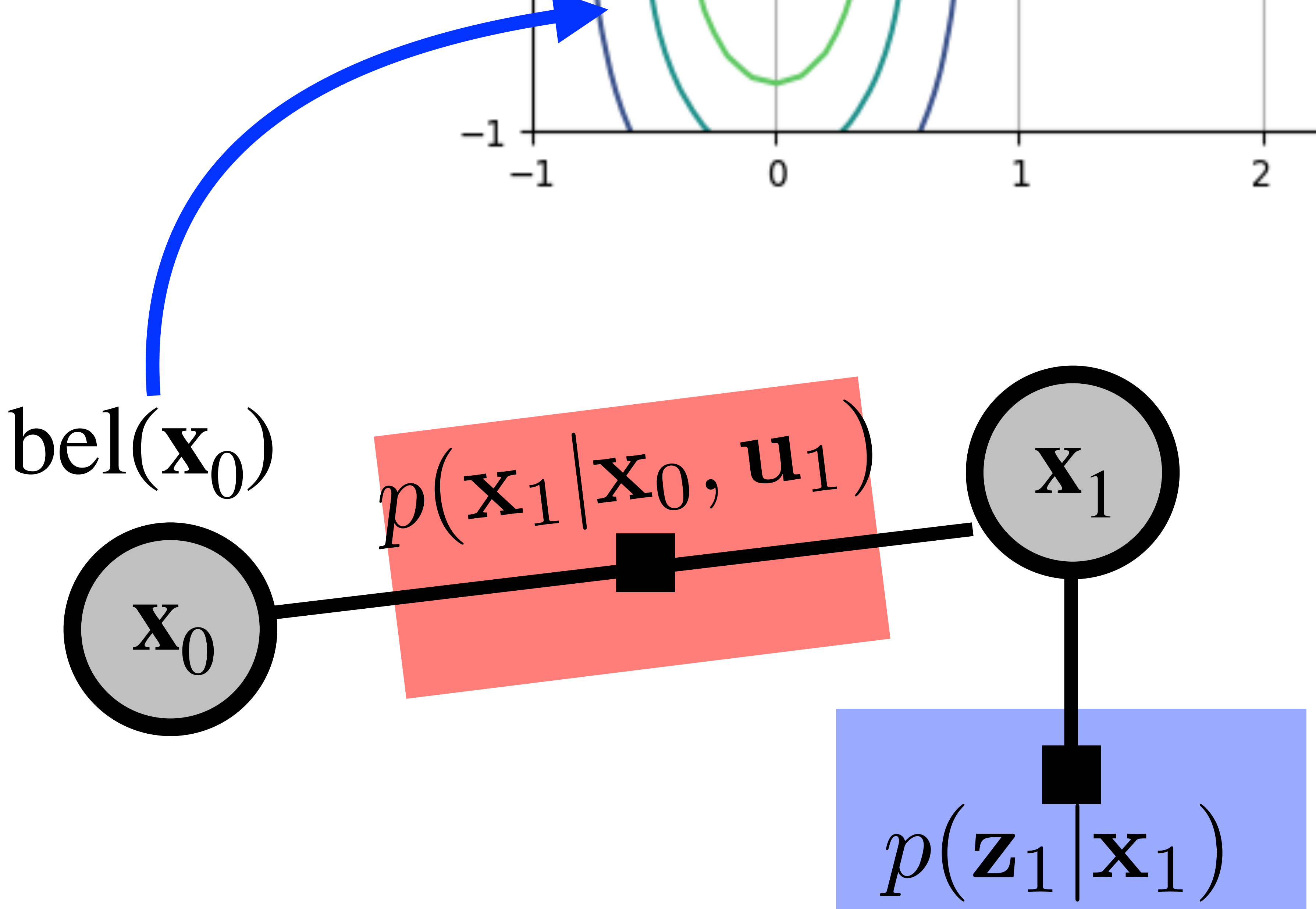
$$p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) = \mathcal{N}\Big(\mathbf{x}_t;\ \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{A}_t}\mathbf{x}_{t-1} + \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{B}_t}\mathbf{u}_t,\ \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{R}_t}\Big)$$

$$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}\Big(\mathbf{z}_t;\ \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{C}_t}\mathbf{x}_{t-1},\ \underbrace{\begin{bmatrix}0.2 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{Q}_t}\Big)$$

1. Initialization:

$$\text{bel}(\mathbf{x}_0) = \mathcal{N}(\mathbf{x}_t;\ \boldsymbol{\mu}_o = \begin{bmatrix}0\\ 0\end{bmatrix}, \boldsymbol{\Sigma}_0 = \begin{bmatrix}0.2 & 0\\ 0 & 1\end{bmatrix})$$





$$p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) = \mathcal{N}\Big(\mathbf{x}_t;\ \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{A}_t}\mathbf{x}_{t-1} + \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{B}_t}\mathbf{u}_t,\ \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{R}_t}\Big)$$

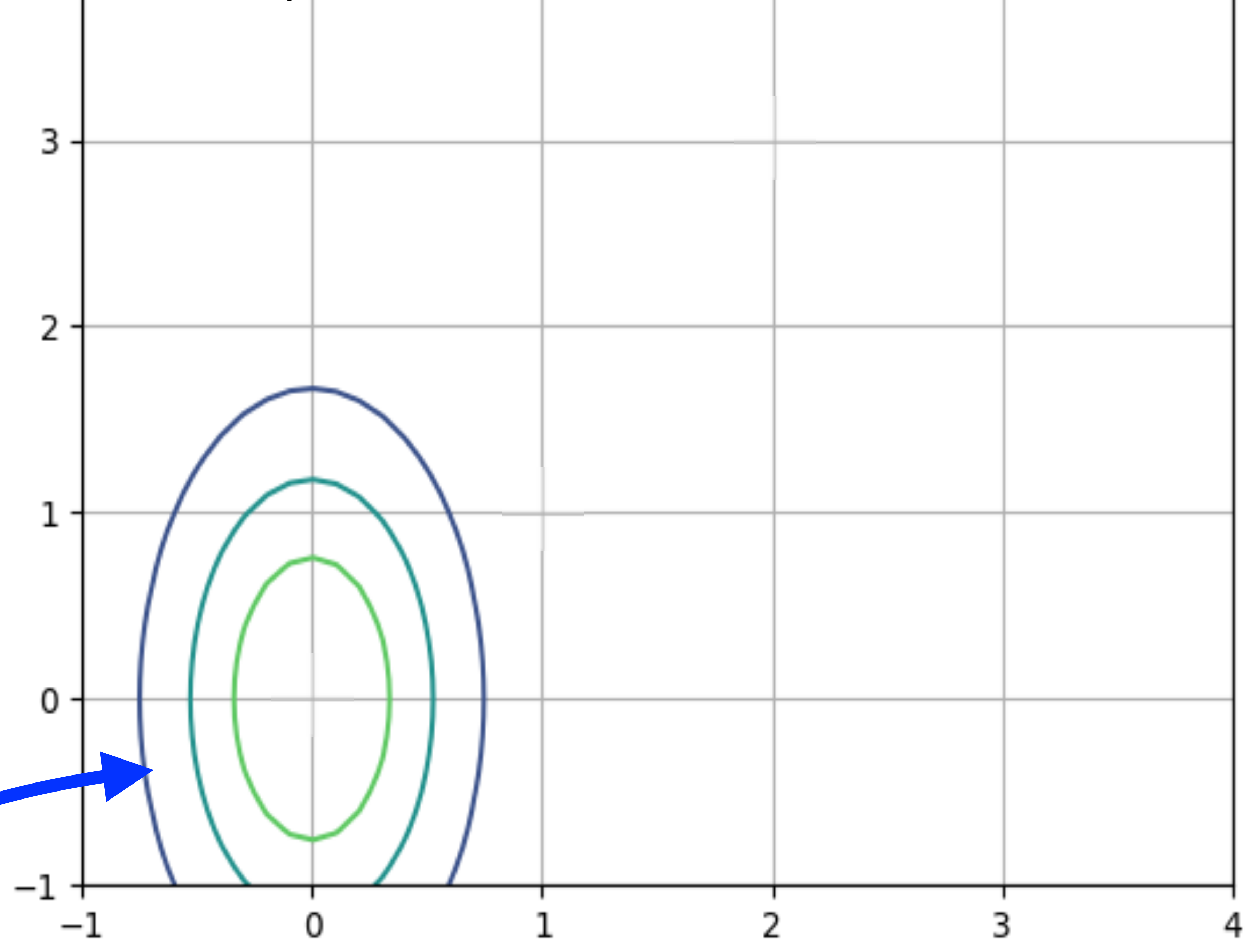
$$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}\Big(\mathbf{z}_t;\ \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{C}_t}\mathbf{x}_{t-1},\ \underbrace{\begin{bmatrix}0.2 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{Q}_t}\Big)$$

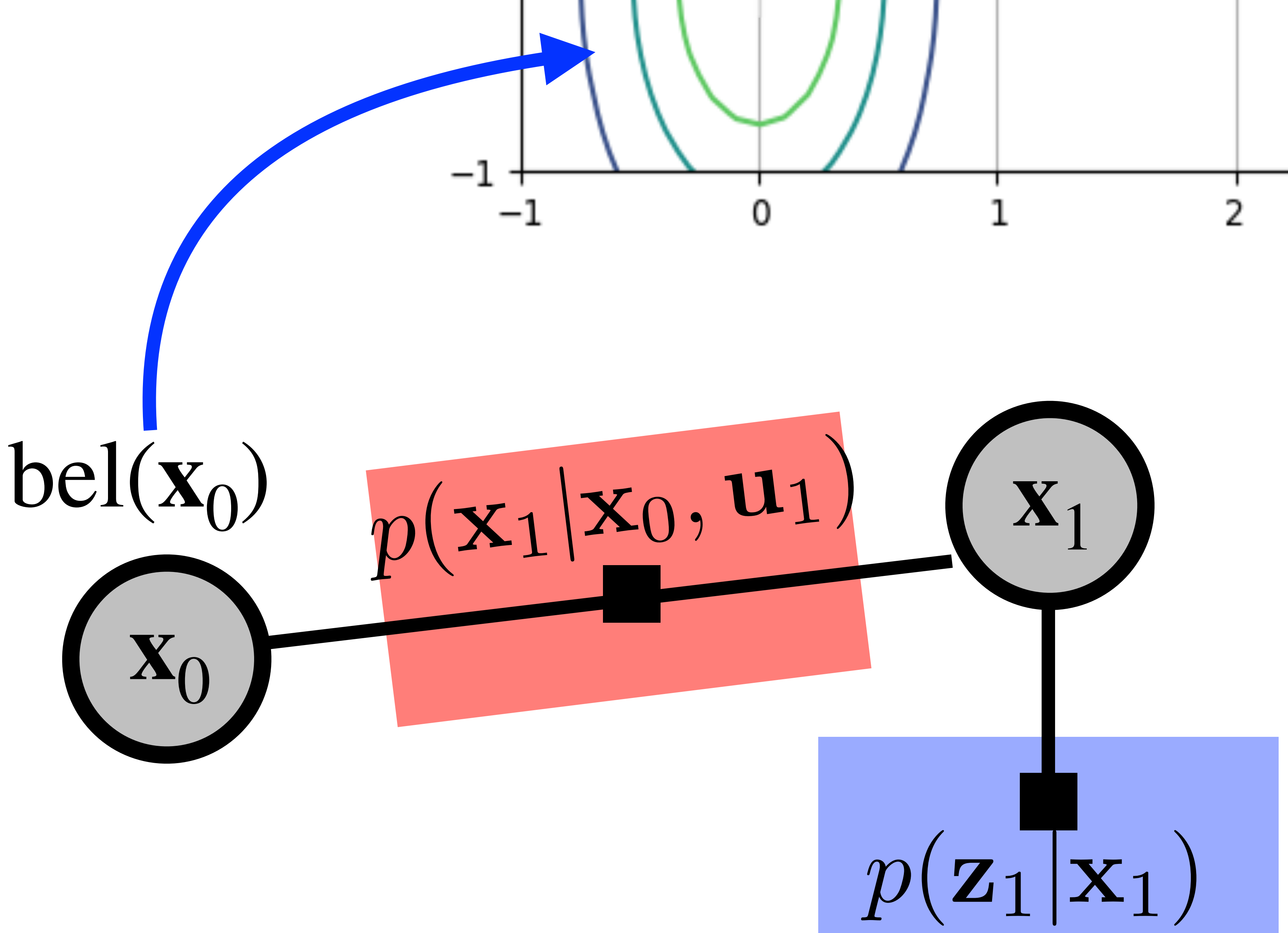
1. Initialization:

$$\text{bel}(\mathbf{x}_0) = \mathcal{N}(\mathbf{x}_t;\ \boldsymbol{\mu}_o = \begin{bmatrix}0\\ 0\end{bmatrix}, \boldsymbol{\Sigma}_0 = \begin{bmatrix}0.2 & 0\\ 0 & 1\end{bmatrix})$$

2. Prediction step (new action $\mathbf{u}_t$):

$$\overline{\text{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$$
$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t\boldsymbol{\mu}_{t-1} + \mathbf{B}_t\mathbf{u}_t$$
$$\overline{\boldsymbol{\Sigma}}_t = \mathbf{A}_t\boldsymbol{\Sigma}_{t-1}\mathbf{A}_t^\top + \mathbf{R}_t$$





$$p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) = \mathcal{N}\Big(\mathbf{x}_t;\ \underbrace{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}}_{\mathbf{A}_t}\mathbf{x}_{t-1} + \underbrace{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}}_{\mathbf{B}_t}\mathbf{u}_t,\ \underbrace{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}}_{\mathbf{R}_t}\Big)$$

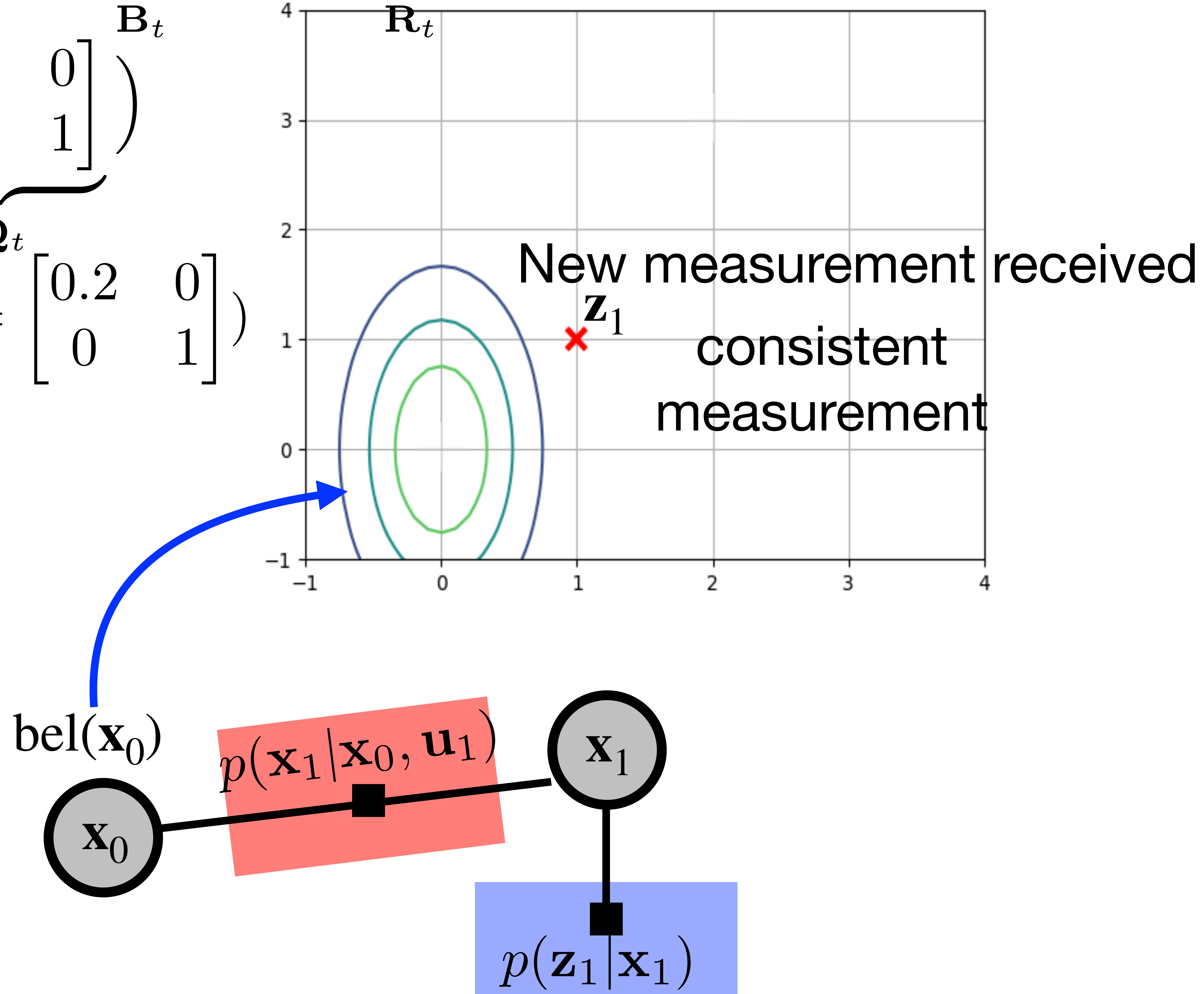
$$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}\Big(\mathbf{z}_t;\ \underbrace{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}}_{\mathbf{C}_t}\mathbf{x}_{t-1},\ \underbrace{\begin{bmatrix}0.2 & 0\\0 & 1\end{bmatrix}}_{\mathbf{Q}_t}\Big)$$

1. Initialization:

   $$\mathrm{bel}(\mathbf{x}_0) = \mathcal{N}\left(\mathbf{x}_t;\ \boldsymbol{\mu}_o = \begin{bmatrix}0\\0\end{bmatrix}, \boldsymbol{\Sigma}_0 = \begin{bmatrix}0.2 & 0\\0 & 1\end{bmatrix}\right)$$

2. Prediction step (new action $\mathbf{u}_t$):

   $$\overline{\mathrm{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$$
   $$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t\boldsymbol{\mu}_{t-1} + \mathbf{B}_t\mathbf{u}_t$$
   $$\overline{\boldsymbol{\Sigma}}_t = \mathbf{A}_t\boldsymbol{\Sigma}_{t-1}\mathbf{A}_t^\top + \mathbf{R}_t$$



$$p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) = \mathcal{N}\Big(\mathbf{x}_t;\ \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{A}_t}\mathbf{x}_{t-1} + \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{B}_t}\mathbf{u}_t,\ \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{R}_t}\Big)$$

$$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}\Big(\mathbf{z}_t;\ \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{C}_t}\mathbf{x}_{t-1},\ \underbrace{\begin{bmatrix}0.2 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{Q}_t}\Big)$$

1. Initialization:

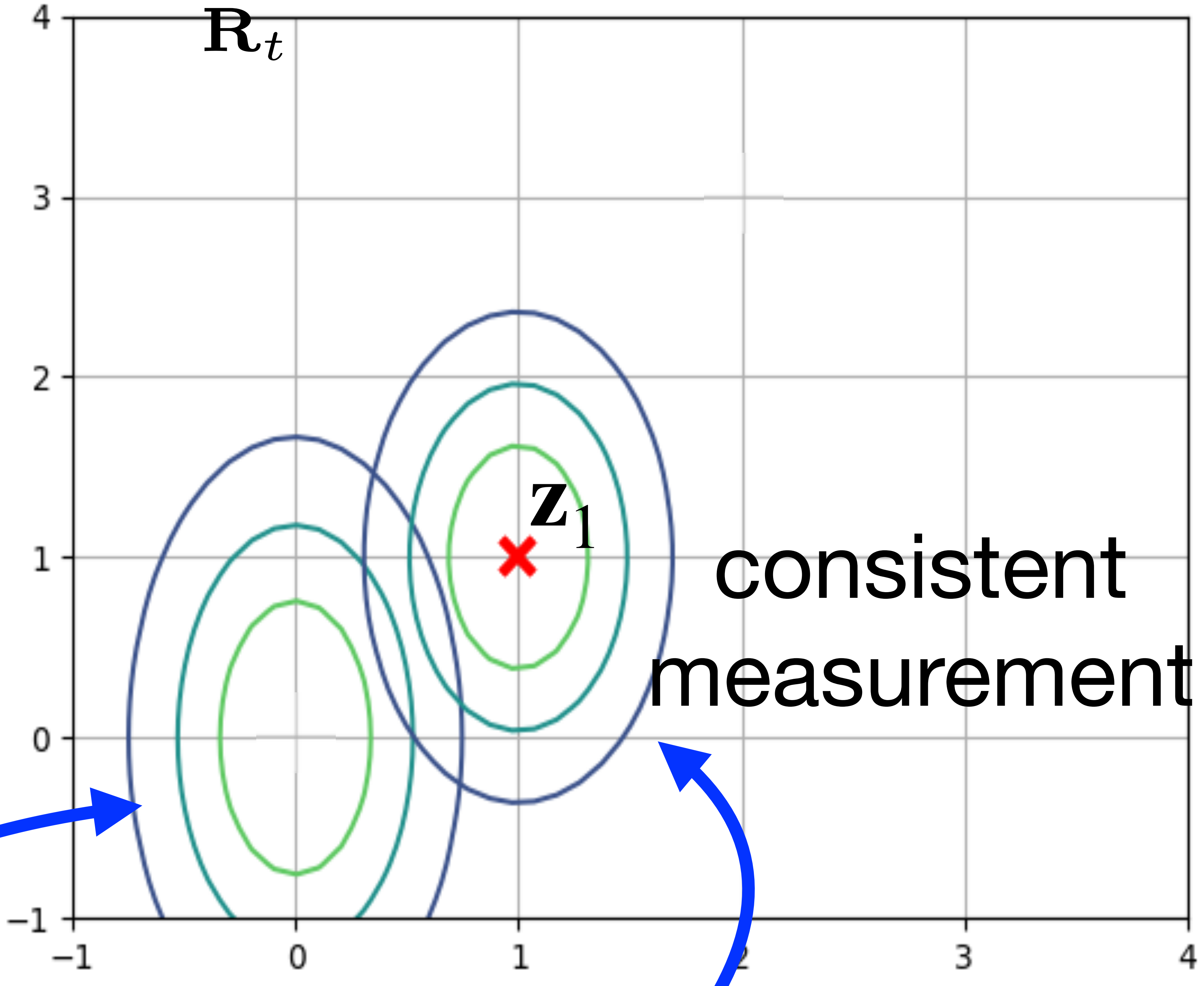
$$\mathrm{bel}(\mathbf{x}_0) = \mathcal{N}\Big(\mathbf{x}_t;\ \boldsymbol{\mu}_o = \begin{bmatrix}0\\ 0\end{bmatrix}, \boldsymbol{\Sigma}_0 = \begin{bmatrix}0.2 & 0\\ 0 & 1\end{bmatrix}\Big)$$

2. Prediction step (new action $\mathbf{u}_t$):

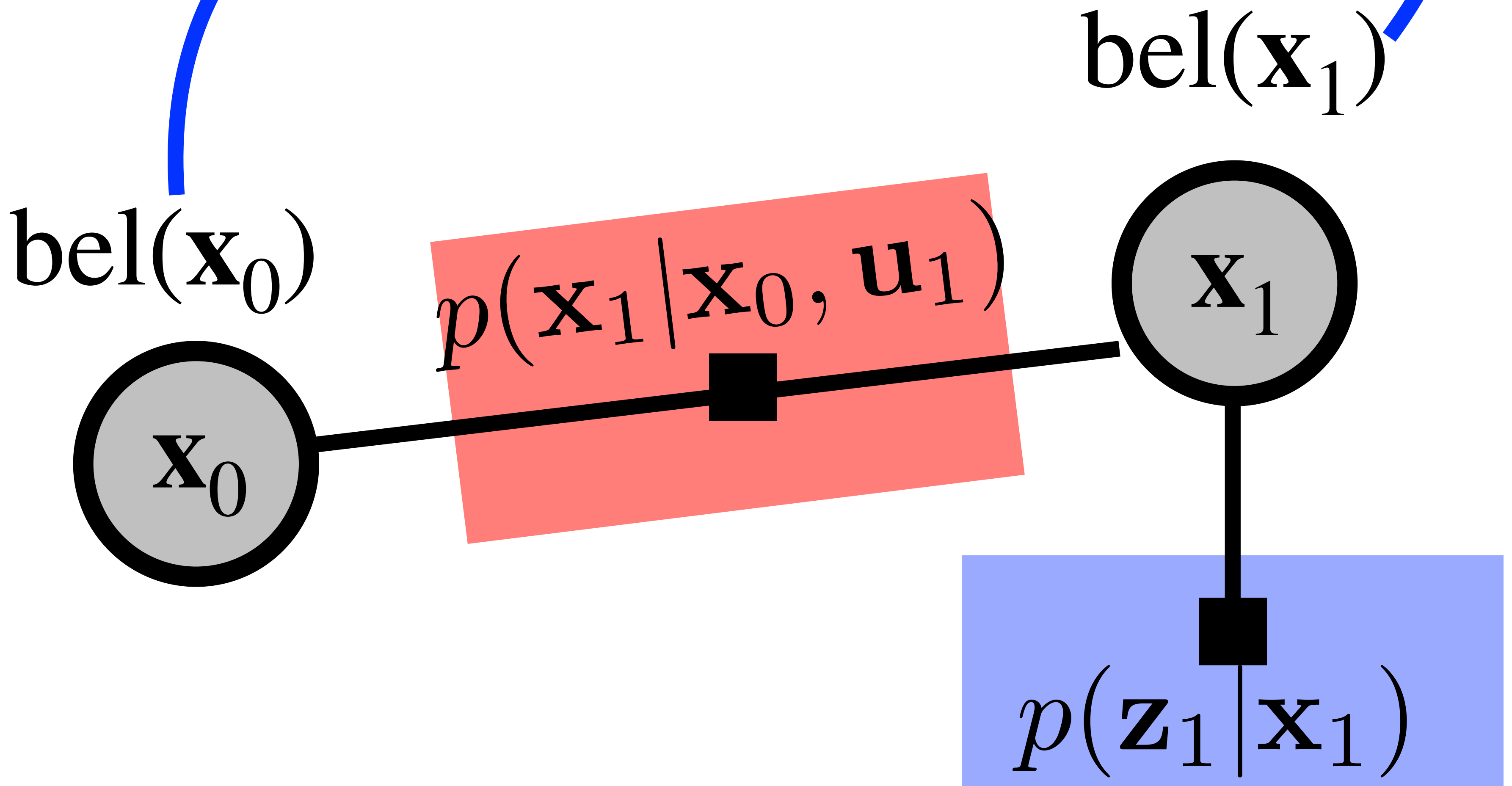
$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$$
$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t\boldsymbol{\mu}_{t-1} + \mathbf{B}_t\mathbf{u}_t$$
$$\overline{\boldsymbol{\Sigma}}_t = \mathbf{A}_t\boldsymbol{\Sigma}_{t-1}\mathbf{A}_t^\top + \mathbf{R}_t$$

3. Measurement update (new $\mathbf{z}_t$):

$$\mathrm{bel}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$$
$$\mathbf{K}_t = \overline{\boldsymbol{\Sigma}}_t\mathbf{C}_t^\top(\mathbf{C}_t\overline{\boldsymbol{\Sigma}}_t\mathbf{C}_t^\top + \mathbf{Q}_t)^{-1}$$
$$\boldsymbol{\mu}_t = \overline{\boldsymbol{\mu}}_t + \mathbf{K}_t(\mathbf{z}_t - \mathbf{C}_t\overline{\boldsymbol{\mu}}_t)$$
$$\boldsymbol{\Sigma}_t = (\mathbf{I} - \mathbf{K}_t\mathbf{C}_t)\overline{\boldsymbol{\Sigma}}_t$$



consistent measurement



Where is the most prob x2?

$$p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) = \mathcal{N}\Big(\mathbf{x}_t;\ \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{A}_t}\mathbf{x}_{t-1} + \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{B}_t}\mathbf{u}_t,\ \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{R}_t}\Big)$$

$$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}\Big(\mathbf{z}_t;\ \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{C}_t}\mathbf{x}_{t-1},\ \underbrace{\begin{bmatrix}0.2 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{Q}_t}\Big)$$

1. Initialization:

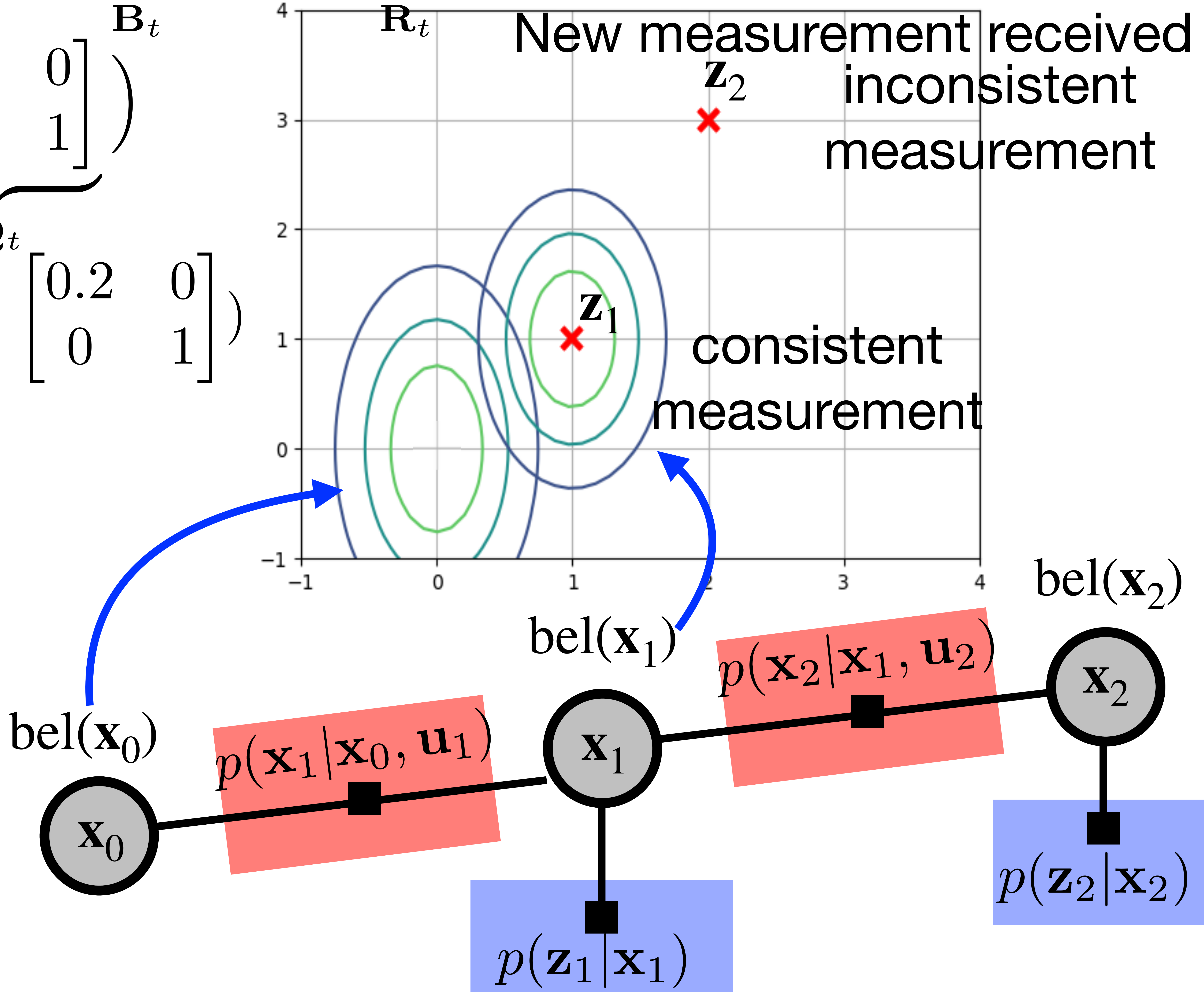
$$\mathrm{bel}(\mathbf{x}_0) = \mathcal{N}\Big(\mathbf{x}_t;\ \boldsymbol{\mu}_o = \begin{bmatrix}0\\ 0\end{bmatrix}, \boldsymbol{\Sigma}_0 = \begin{bmatrix}0.2 & 0\\ 0 & 1\end{bmatrix}\Big)$$

2. Prediction step (new action $\mathbf{u}_t$):

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$$
$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t\boldsymbol{\mu}_{t-1} + \mathbf{B}_t\mathbf{u}_t$$
$$\overline{\boldsymbol{\Sigma}}_t = \mathbf{A}_t\boldsymbol{\Sigma}_{t-1}\mathbf{A}_t^\top + \mathbf{R}_t$$

3. Measurement update (new $\mathbf{z}_t$):

$$\mathrm{bel}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$$
$$\mathbf{K}_t = \overline{\boldsymbol{\Sigma}}_t\mathbf{C}_t^\top(\mathbf{C}_t\overline{\boldsymbol{\Sigma}}_t\mathbf{C}_t^\top + \mathbf{Q}_t)^{-1}$$
$$\boldsymbol{\mu}_t = \overline{\boldsymbol{\mu}}_t + \mathbf{K}_t(\mathbf{z}_t - \mathbf{C}_t\overline{\boldsymbol{\mu}}_t)$$
$$\boldsymbol{\Sigma}_t = (\mathbf{I} - \mathbf{K}_t\mathbf{C}_t)\overline{\boldsymbol{\Sigma}}_t$$



$$p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t)=\mathcal{N}\Big(\mathbf{x}_t;\ \underbrace{\begin{bmatrix}1&0\\0&1\end{bmatrix}}_{\mathbf{A}_t}\mathbf{x}_{t-1}+\underbrace{\begin{bmatrix}1&0\\0&1\end{bmatrix}}_{\mathbf{B}_t}\mathbf{u}_t,\ \underbrace{\begin{bmatrix}1&0\\0&1\end{bmatrix}}_{\mathbf{R}_t}\Big)$$

$$p(\mathbf{z}_t|\mathbf{x}_t)=\mathcal{N}\Big(\mathbf{z}_t;\ \underbrace{\begin{bmatrix}1&0\\0&1\end{bmatrix}}_{\mathbf{C}_t}\mathbf{x}_{t-1},\ \underbrace{\begin{bmatrix}0.2&0\\0&1\end{bmatrix}}_{\mathbf{Q}_t}\Big)$$

1. Initialization:

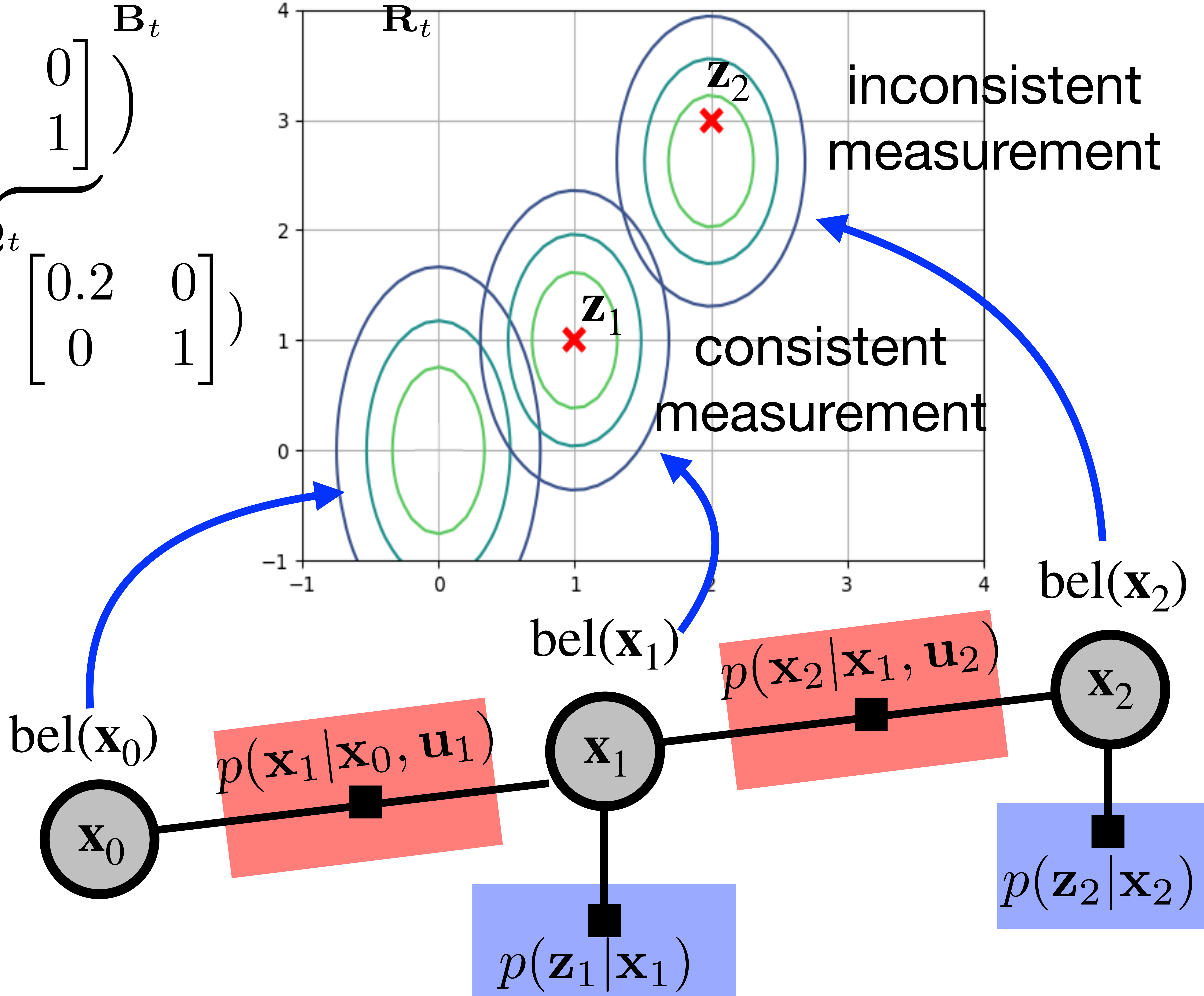
$$\mathrm{bel}(\mathbf{x}_0)=\mathcal{N}(\mathbf{x}_t;\ \boldsymbol{\mu}_o=\begin{bmatrix}0\\0\end{bmatrix},\boldsymbol{\Sigma}_0=\begin{bmatrix}0.2&0\\0&1\end{bmatrix})$$

2. Prediction step (new action $\mathbf{u}_t$):

$$\overline{\mathrm{bel}}(\mathbf{x}_t)=\mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t,\overline{\boldsymbol{\Sigma}}_t)$$
$$\overline{\boldsymbol{\mu}}_t=\mathbf{A}_t\boldsymbol{\mu}_{t-1}+\mathbf{B}_t\mathbf{u}_t$$
$$\overline{\boldsymbol{\Sigma}}_t=\mathbf{A}_t\boldsymbol{\Sigma}_{t-1}\mathbf{A}_t^\top+\mathbf{R}_t$$

3. Measurement update (new $\mathbf{z}_t$):

$$\mathrm{bel}(\mathbf{x}_t)=\mathcal{N}_{\mathbf{x}_t}(\boldsymbol{\mu}_t,\boldsymbol{\Sigma}_t)$$
$$\mathbf{K}_t=\overline{\boldsymbol{\Sigma}}_t\mathbf{C}_t^\top(\mathbf{C}_t\overline{\boldsymbol{\Sigma}}_t\mathbf{C}_t^\top+\mathbf{Q}_t)^{-1}$$
$$\boldsymbol{\mu}_t=\overline{\boldsymbol{\mu}}_t+\mathbf{K}_t(\mathbf{z}_t-\mathbf{C}_t\overline{\boldsymbol{\mu}}_t)$$
$$\boldsymbol{\Sigma}_t=(\mathbf{I}-\mathbf{K}_t\mathbf{C}_t)\overline{\boldsymbol{\Sigma}}_t$$



$$p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) = \mathcal{N}\Big(\mathbf{x}_t;\ \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{A}_t}\mathbf{x}_{t-1} + \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{B}_t}\mathbf{u}_t,\ \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{R}_t}\Big)$$

$$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}\Big(\mathbf{z}_t;\ \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{C}_t}\mathbf{x}_{t-1},\ \underbrace{\begin{bmatrix}0.2 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{Q}_t}\Big)$$

1. Initialization:

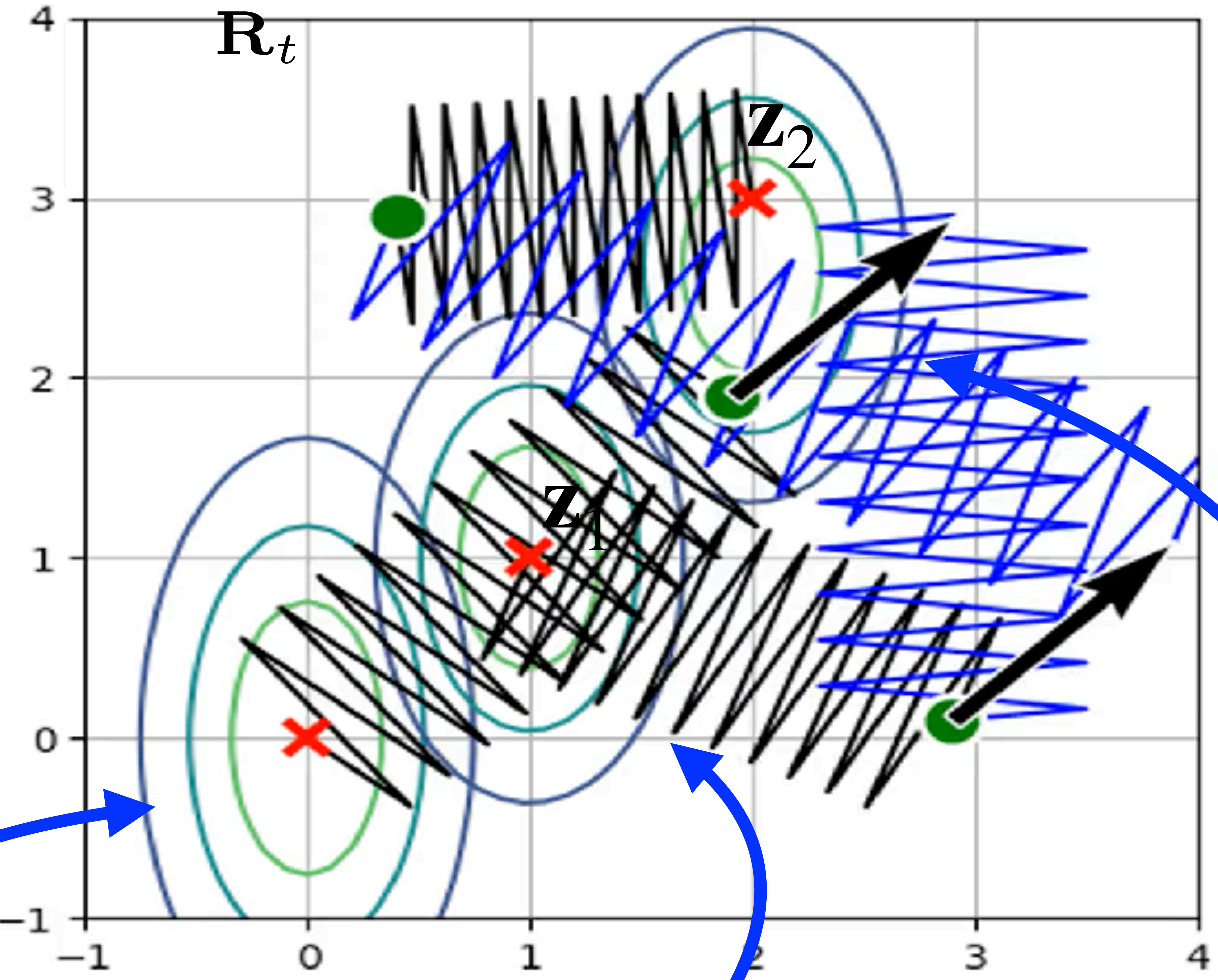
$$\mathrm{bel}(\mathbf{x}_0) = \mathcal{N}(\mathbf{x}_t;\ \boldsymbol{\mu}_o = \begin{bmatrix}0\\ 0\end{bmatrix}, \boldsymbol{\Sigma}_0 = \begin{bmatrix}0.2 & 0\\ 0 & 1\end{bmatrix})$$

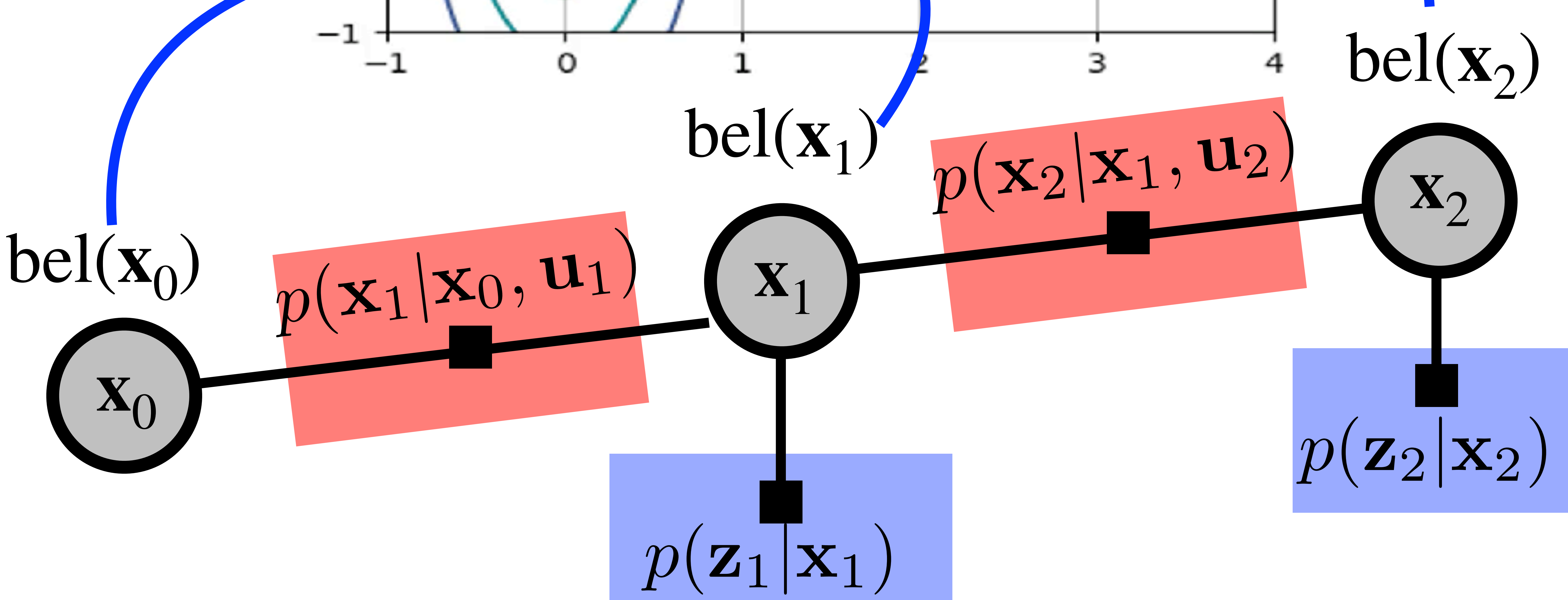
2. Prediction step (new action $\mathbf{u}_t$):

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$$
$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t\boldsymbol{\mu}_{t-1} + \mathbf{B}_t\mathbf{u}_t$$
$$\overline{\boldsymbol{\Sigma}}_t = \mathbf{A}_t\boldsymbol{\Sigma}_{t-1}\mathbf{A}_t^\top + \mathbf{R}_t$$

3. Measurement update (new $\mathbf{z}_t$):

$$\mathrm{bel}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$$
$$\mathbf{K}_t = \overline{\boldsymbol{\Sigma}}_t\mathbf{C}_t^\top(\mathbf{C}_t\overline{\boldsymbol{\Sigma}}_t\mathbf{C}_t^\top + \mathbf{Q}_t)^{-1}$$
$$\boldsymbol{\mu}_t = \overline{\boldsymbol{\mu}}_t + \mathbf{K}_t(\mathbf{z}_t - \mathbf{C}_t\overline{\boldsymbol{\mu}}_t)$$
$$\boldsymbol{\Sigma}_t = (\mathbf{I} - \mathbf{K}_t\mathbf{C}_t)\overline{\boldsymbol{\Sigma}}_t$$





KF example: state = (x … position, v …velocity)

$$p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) = \mathcal{N}_{\mathbf{x}_t}\Big(\underbrace{\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}}_{\mathbf{A}_t}\mathbf{x}_{t-1} + \underbrace{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}}_{\mathbf{B}_t}\mathbf{u}_t,\ \underbrace{\begin{bmatrix}0.01 & 0\\0 & 0.01\end{bmatrix}}_{\mathbf{R}_t}\Big)$$

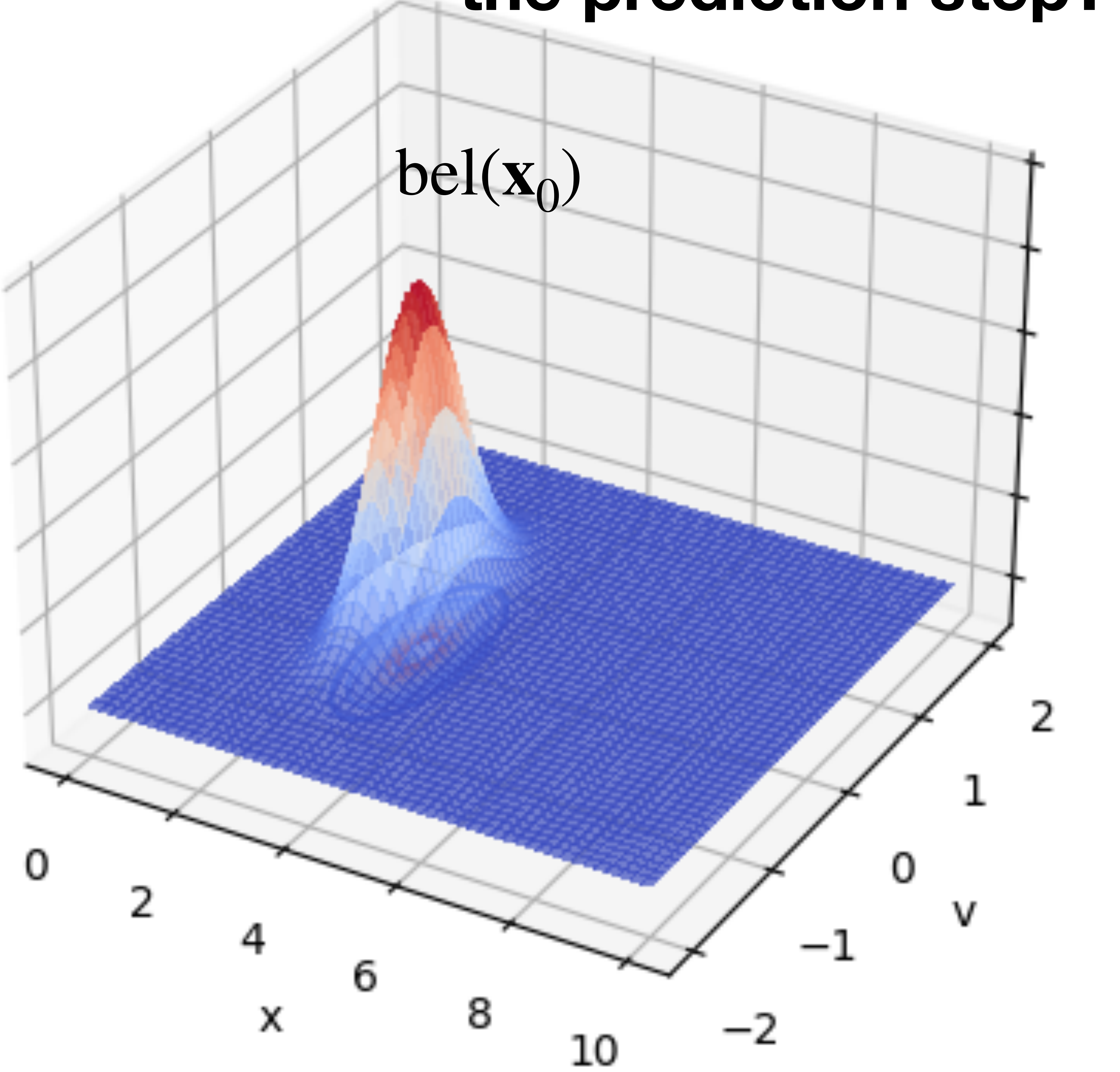
$$\mathbf{u}_t = \begin{bmatrix}0\\0\end{bmatrix}$$

What happens after the prediction step?

$$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}_{\mathbf{z}_t}\Big(\underbrace{\begin{bmatrix}1 & 0\end{bmatrix}}_{\mathbf{C}_t}\mathbf{x}_{t-1},\ \underbrace{\begin{bmatrix}0.3\end{bmatrix}}_{\mathbf{Q}_t}\Big)$$

2. Prediction step (new action $\mathbf{u}_t$ performed):

$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t\boldsymbol{\mu}_{t-1} + \mathbf{B}_t\mathbf{u}_t$$
$$\overline{\boldsymbol{\Sigma}}_t = \mathbf{A}_t\boldsymbol{\Sigma}_{t-1}\mathbf{A}_t^\top + \mathbf{R}_t$$
$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$$



KF example: state = (x … position, v …velocity)

$$p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) = \mathcal{N}_{\mathbf{x}_t}\Big(\underbrace{\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}}_{\mathbf{A}_t}\mathbf{x}_{t-1} + \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{B}_t}\mathbf{u}_t,\ \underbrace{\begin{bmatrix}0.01 & 0\\ 0 & 0.01\end{bmatrix}}_{\mathbf{R}_t}\Big)$$

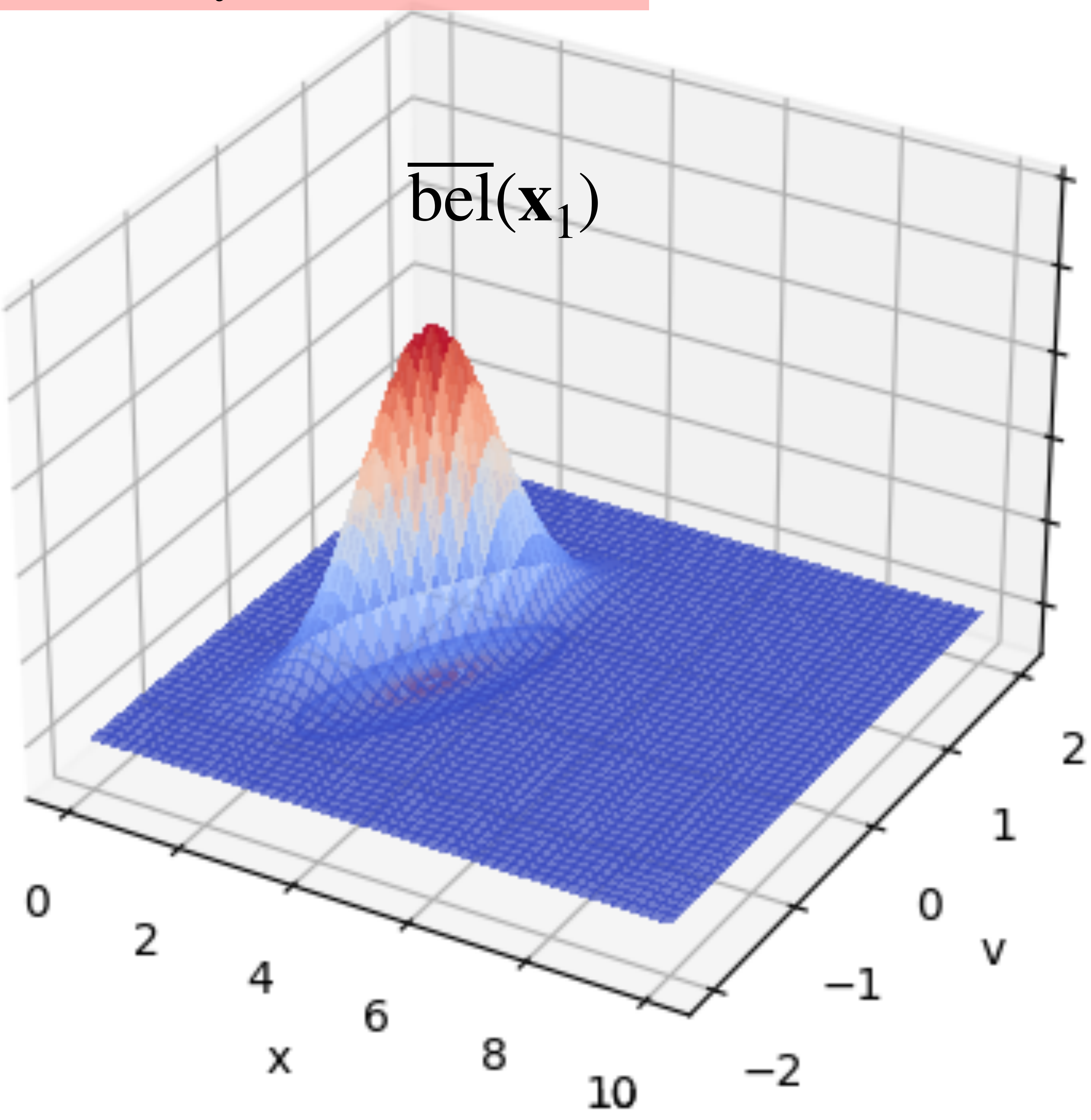
$$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}_{\mathbf{z}_t}\Big(\underbrace{\begin{bmatrix}1 & 0\end{bmatrix}}_{\mathbf{C}_t}\mathbf{x}_{t-1},\ \underbrace{\begin{bmatrix}0.3\end{bmatrix}}_{\mathbf{Q}_t}\Big)$$

2. Prediction step (new action $\mathbf{u}_t$ performed):

$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t\boldsymbol{\mu}_{t-1} + \mathbf{B}_t\mathbf{u}_t$$

$$\overline{\boldsymbol{\Sigma}}_t = \mathbf{A}_t\boldsymbol{\Sigma}_{t-1}\mathbf{A}_t^\top + \mathbf{R}_t$$

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$$



KF example: state = (x … position, v …velocity)

$$p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) = \mathcal{N}_{\mathbf{x}_t}\Bigg(\underbrace{\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}}_{\mathbf{A}_t}\mathbf{x}_{t-1} + \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{B}_t}\mathbf{u}_t,\ \underbrace{\begin{bmatrix}0.01 & 0\\ 0 & 0.01\end{bmatrix}}_{\mathbf{R}_t}\Bigg)$$

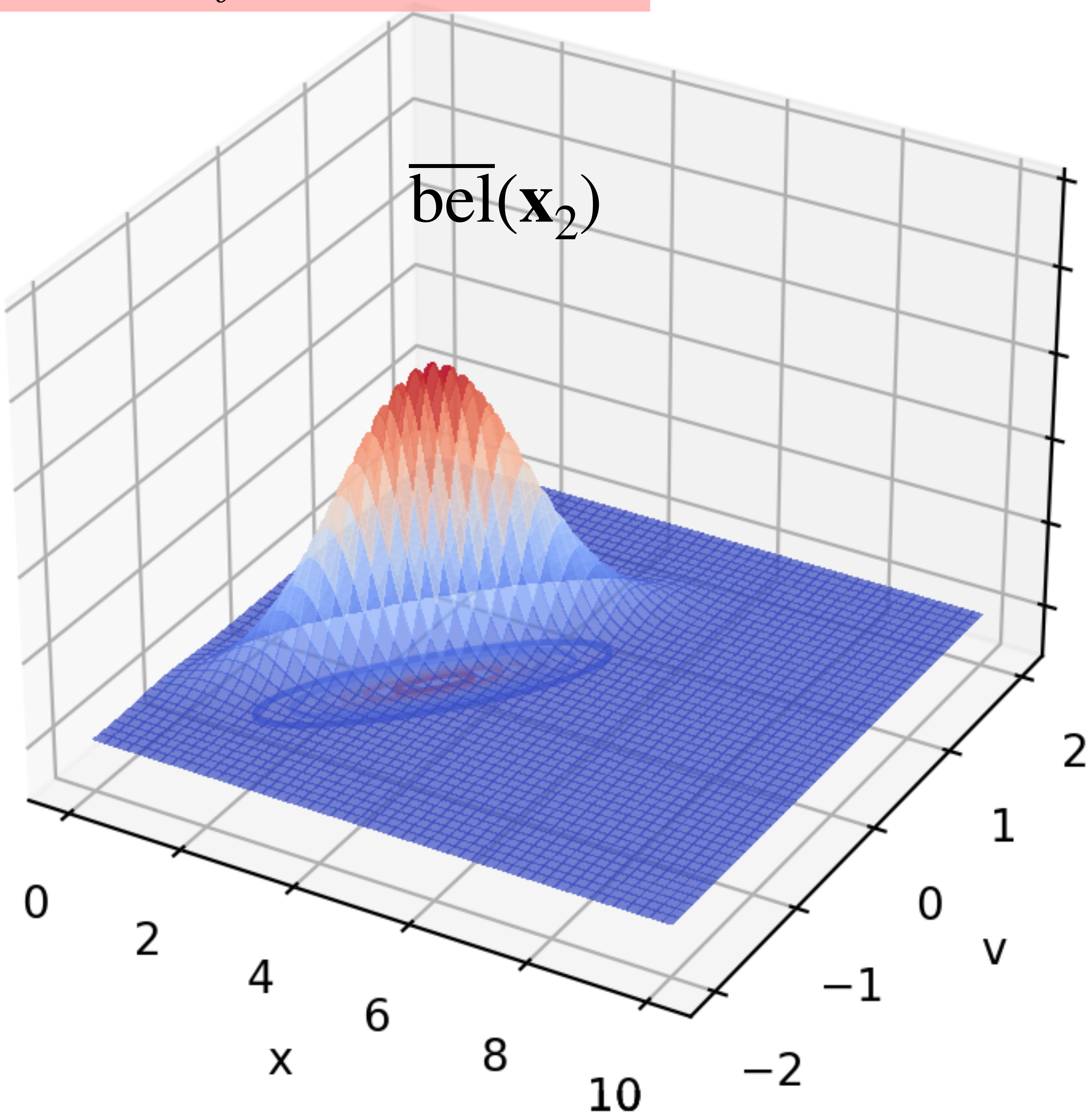
$$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}_{\mathbf{z}_t}\Bigg(\underbrace{\begin{bmatrix}1 & 0\end{bmatrix}}_{\mathbf{C}_t}\mathbf{x}_{t-1},\ \underbrace{\begin{bmatrix}0.3\end{bmatrix}}_{\mathbf{Q}_t}\Bigg)$$

2. Prediction step (new action $\mathbf{u}_t$ performed):

$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t\boldsymbol{\mu}_{t-1} + \mathbf{B}_t\mathbf{u}_t$$

$$\overline{\boldsymbol{\Sigma}}_t = \mathbf{A}_t\boldsymbol{\Sigma}_{t-1}\mathbf{A}_t^\top + \mathbf{R}_t$$

$$\overline{\text{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$$



KF example: state = (x … position, v …velocity)

$$p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) = \mathcal{N}_{\mathbf{x}_t}\Big(\underbrace{\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}}_{\mathbf{A}_t}\mathbf{x}_{t-1} + \underbrace{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}}_{\mathbf{B}_t}\mathbf{u}_t,\ \underbrace{\begin{bmatrix}0.01 & 0\\0 & 0.01\end{bmatrix}}_{\mathbf{R}_t}\Big)$$

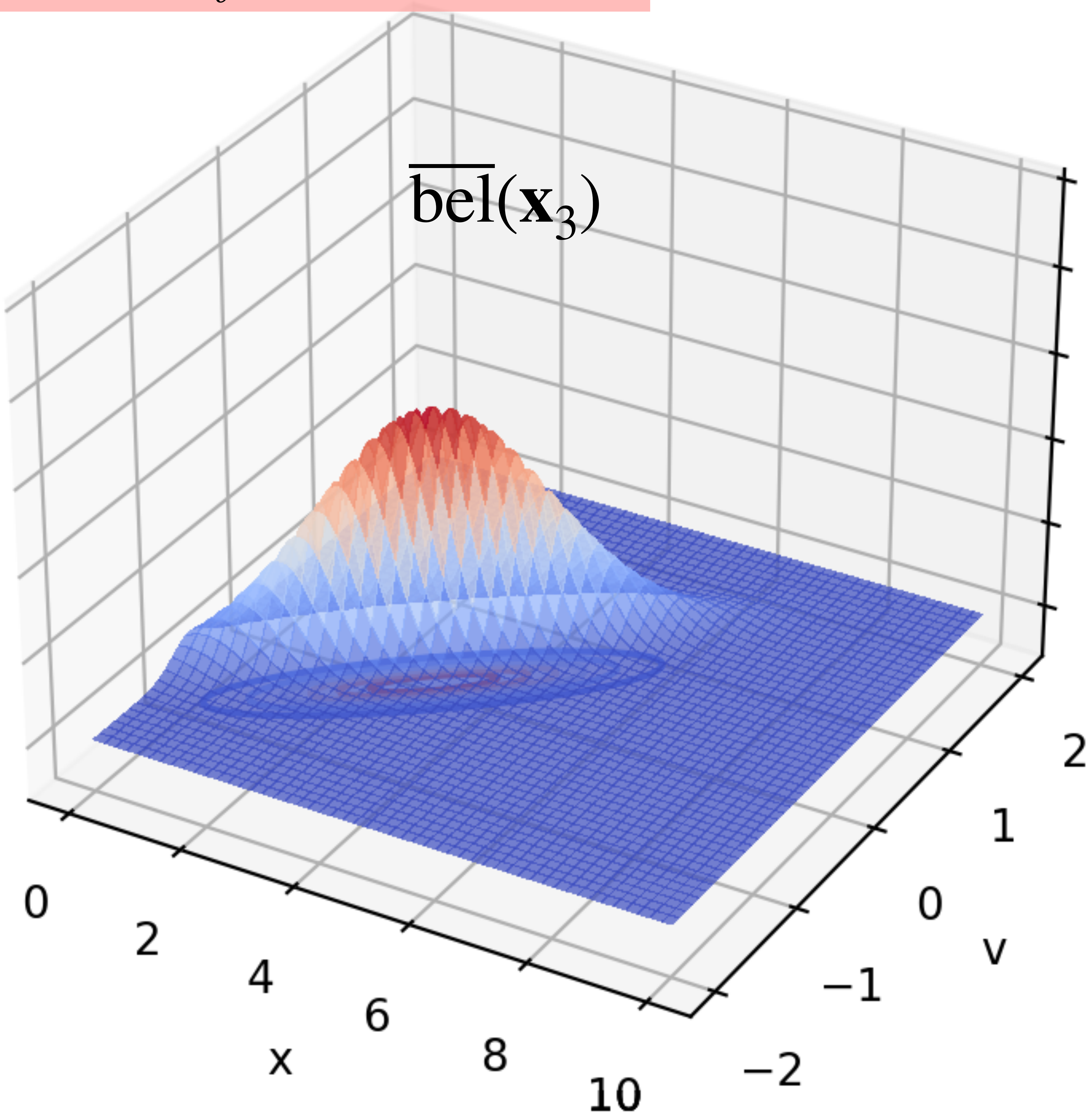
$$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}_{\mathbf{z}_t}\Big(\underbrace{\begin{bmatrix}1 & 0\end{bmatrix}}_{\mathbf{C}_t}\mathbf{x}_{t-1},\ \underbrace{\begin{bmatrix}0.3\end{bmatrix}}_{\mathbf{Q}_t}\Big)$$

2. Prediction step (new action $\mathbf{u}_t$ performed):

$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t\boldsymbol{\mu}_{t-1} + \mathbf{B}_t\mathbf{u}_t$$

$$\overline{\boldsymbol{\Sigma}}_t = \mathbf{A}_t\boldsymbol{\Sigma}_{t-1}\mathbf{A}_t^\top + \mathbf{R}_t$$

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$$



KF example: state = (x … position, v …velocity)

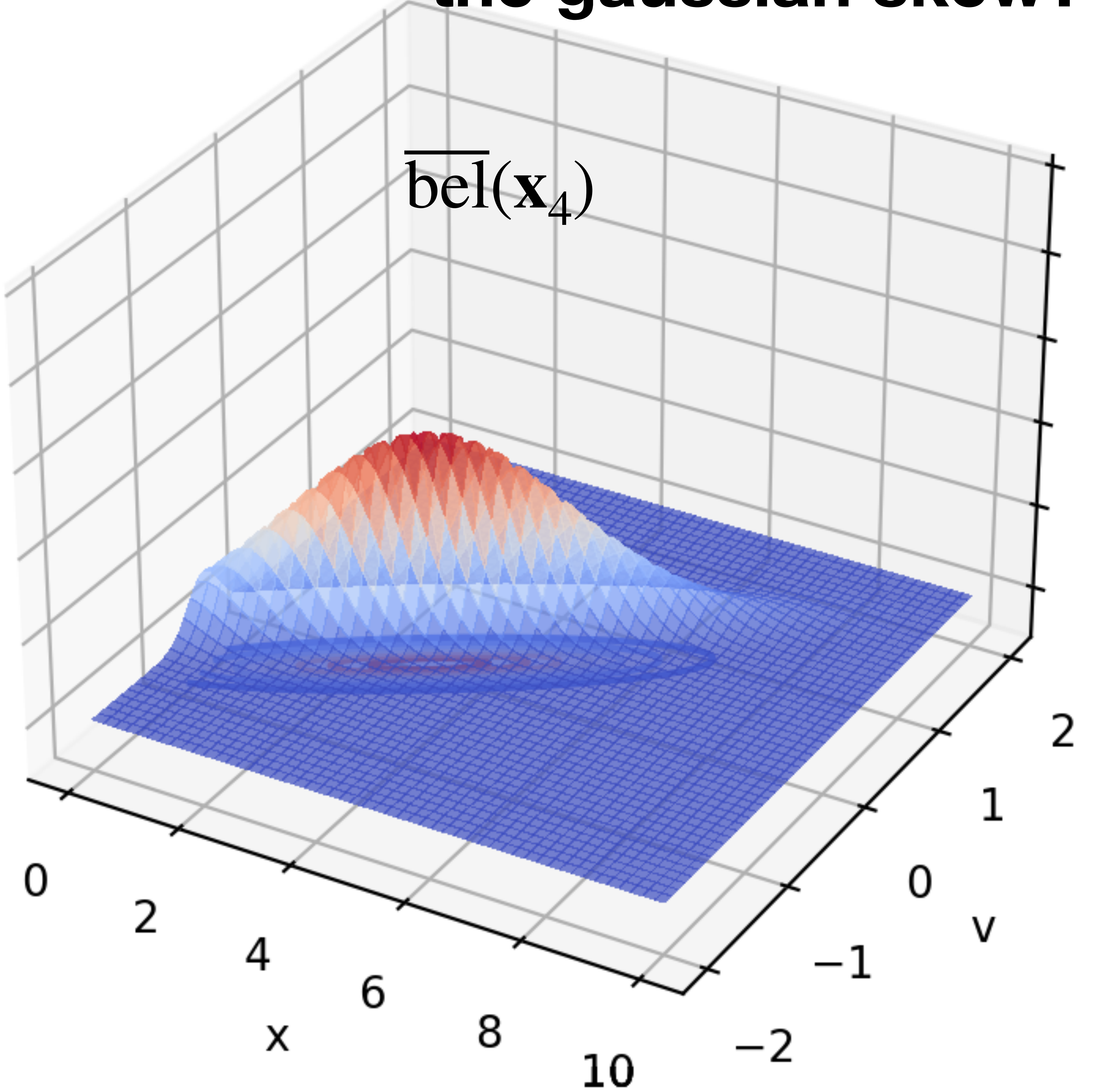
$$p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) = \mathcal{N}_{\mathbf{x}_t}\Big(\underbrace{\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}}_{\mathbf{A}_t}\mathbf{x}_{t-1} + \underbrace{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}}_{\mathbf{B}_t}\mathbf{u}_t,\ \underbrace{\begin{bmatrix}0.01 & 0\\0 & 0.01\end{bmatrix}}_{\mathbf{R}_t}\Big)$$

$$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}_{\mathbf{z}_t}\Big(\underbrace{\begin{bmatrix}1 & 0\end{bmatrix}}_{\mathbf{C}_t}\mathbf{x}_{t-1},\ \underbrace{\begin{bmatrix}0.3\end{bmatrix}}_{\mathbf{Q}_t}\Big)$$

Can you explain the gaussian skew?

2. Prediction step (new action $\mathbf{u}_t$ performed):

$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t\boldsymbol{\mu}_{t-1} + \mathbf{B}_t\mathbf{u}_t$$
$$\overline{\boldsymbol{\Sigma}}_t = \mathbf{A}_t\boldsymbol{\Sigma}_{t-1}\mathbf{A}_t^\top + \mathbf{R}_t$$
$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$$



KF example: state = (x … position, v …velocity)

$$p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) = \mathcal{N}_{\mathbf{x}_t}\Big(\underbrace{\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}}_{\mathbf{A}_t}\mathbf{x}_{t-1} + \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{B}_t}\mathbf{u}_t,\ \underbrace{\begin{bmatrix}0.01 & 0\\ 0 & 0.01\end{bmatrix}}_{\mathbf{R}_t}\Big)$$

Position measurement

$$\mathbf{z}_5 = 5$$

What happens after the measurement step?

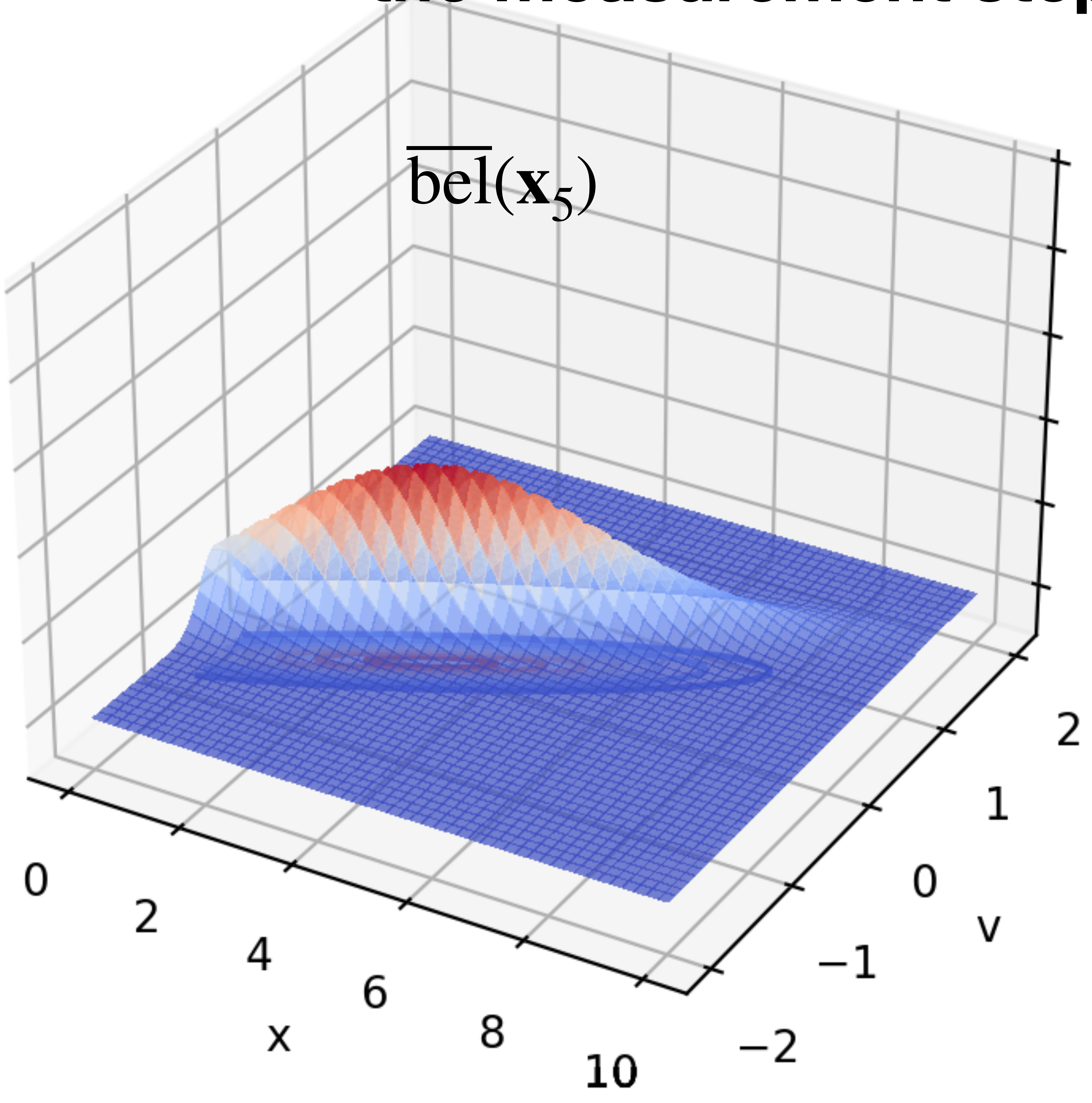
$$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}_{\mathbf{z}_t}\Big(\underbrace{\begin{bmatrix}1 & 0\end{bmatrix}}_{\mathbf{C}_t}\mathbf{x}_{t-1},\ \underbrace{\begin{bmatrix}0.3\end{bmatrix}}_{\mathbf{Q}_t}\Big)$$

2. Prediction step (new action $\mathbf{u}_t$ performed):

$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t\boldsymbol{\mu}_{t-1} + \mathbf{B}_t\mathbf{u}_t$$
$$\overline{\boldsymbol{\Sigma}}_t = \mathbf{A}_t\boldsymbol{\Sigma}_{t-1}\mathbf{A}_t^\top + \mathbf{R}_t$$
$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$$

3. Measurement update (new $\mathbf{z}_t$ received):

$$\mathbf{K}_t = \overline{\boldsymbol{\Sigma}}_t\mathbf{C}_t^\top(\mathbf{C}_t\overline{\boldsymbol{\Sigma}}_t\mathbf{C}_t^\top + \mathbf{Q}_t)^{-1}$$
$$\boldsymbol{\mu}_t = \overline{\boldsymbol{\mu}}_t + \mathbf{K}_t(\mathbf{z}_t - \mathbf{C}_t\overline{\boldsymbol{\mu}}_t)$$
$$\boldsymbol{\Sigma}_t = (\mathbf{I} - \mathbf{K}_t\mathbf{C}_t)\overline{\boldsymbol{\Sigma}}_t$$
$$\mathrm{bel}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$$



KF example: state = (x … position, v …velocity)

$$p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) = \mathcal{N}_{\mathbf{x}_t}\Big(\underbrace{\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}}_{\mathbf{A}_t}\mathbf{x}_{t-1} + \underbrace{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}}_{\mathbf{B}_t}\mathbf{u}_t,\ \underbrace{\begin{bmatrix}0.01 & 0\\0 & 0.01\end{bmatrix}}_{\mathbf{R}_t}\Big)$$

$$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}_{\mathbf{z}_t}\Big(\underbrace{\begin{bmatrix}1 & 0\end{bmatrix}}_{\mathbf{C}_t}\mathbf{x}_{t-1},\ \underbrace{\begin{bmatrix}0.3\end{bmatrix}}_{\mathbf{Q}_t}\Big)$$

$\mathbf{z}_5 = 5$

Velocity = (x5-x0)/5

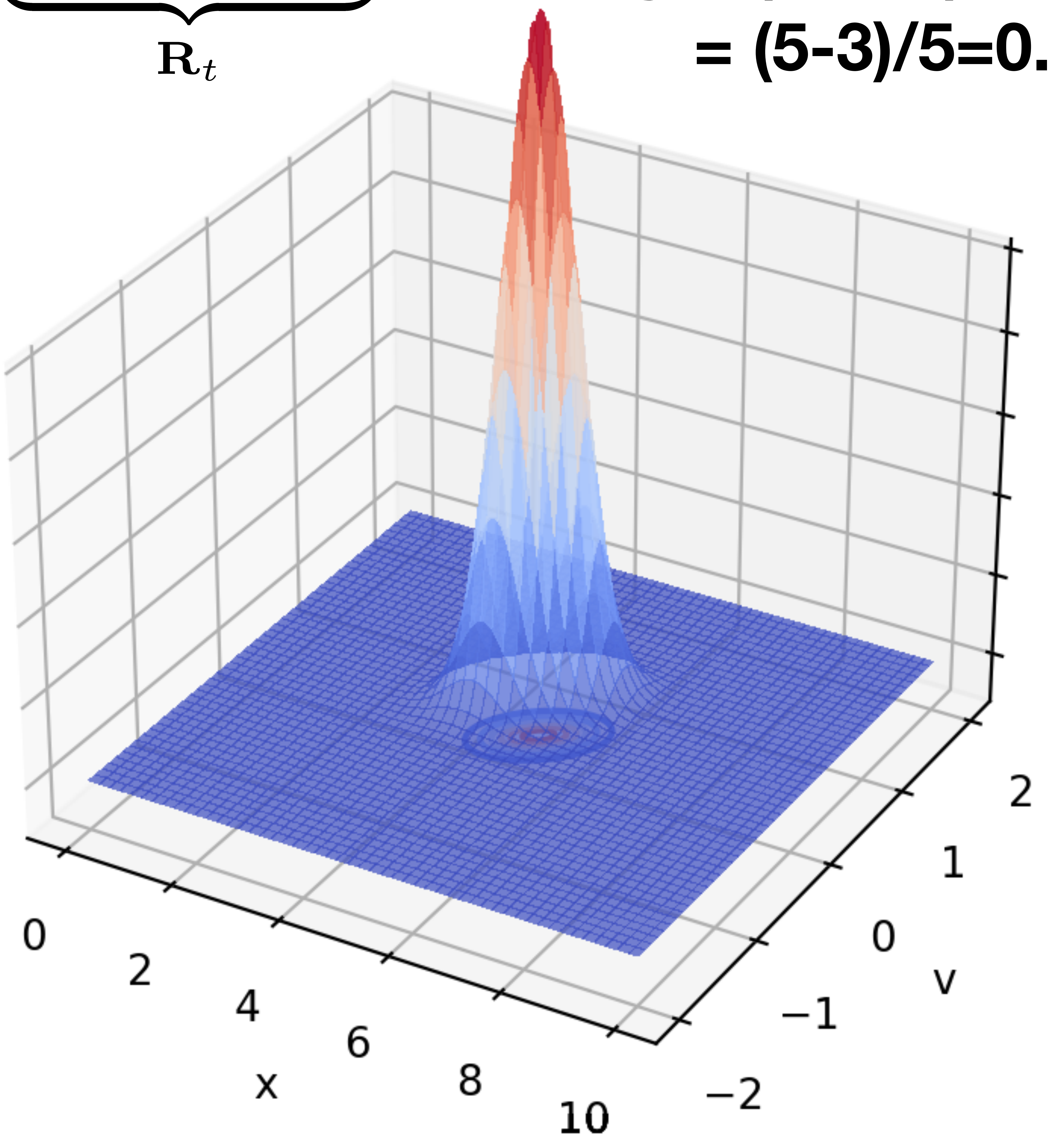
= (5-3)/5=0.4

2. Prediction step (new action $\mathbf{u}_t$ performed):

$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t\boldsymbol{\mu}_{t-1} + \mathbf{B}_t\mathbf{u}_t$$
$$\overline{\boldsymbol{\Sigma}}_t = \mathbf{A}_t\boldsymbol{\Sigma}_{t-1}\mathbf{A}_t^\top + \mathbf{R}_t$$
$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$$

3. Measurement update (new $\mathbf{z}_t$ received):

$$\mathbf{K}_t = \overline{\boldsymbol{\Sigma}}_t\mathbf{C}_t^\top(\mathbf{C}_t\overline{\boldsymbol{\Sigma}}_t\mathbf{C}_t^\top + \mathbf{Q}_t)^{-1}$$
$$\boldsymbol{\mu}_t = \overline{\boldsymbol{\mu}}_t + \mathbf{K}_t(\mathbf{z}_t - \mathbf{C}_t\overline{\boldsymbol{\mu}}_t)$$
$$\boldsymbol{\Sigma}_t = (\mathbf{I} - \mathbf{K}_t\mathbf{C}_t)\overline{\boldsymbol{\Sigma}}_t$$
$$\mathrm{bel}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$$



KF example: state = (x … position, v …velocity)

$$p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) = \mathcal{N}_{\mathbf{x}_t}\Big(\underbrace{\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}}_{\mathbf{A}_t}\mathbf{x}_{t-1} + \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{B}_t}\mathbf{u}_t,\ \underbrace{\begin{bmatrix}0.01 & 0\\ 0 & 0.01\end{bmatrix}}_{\mathbf{R}_t}\Big)$$

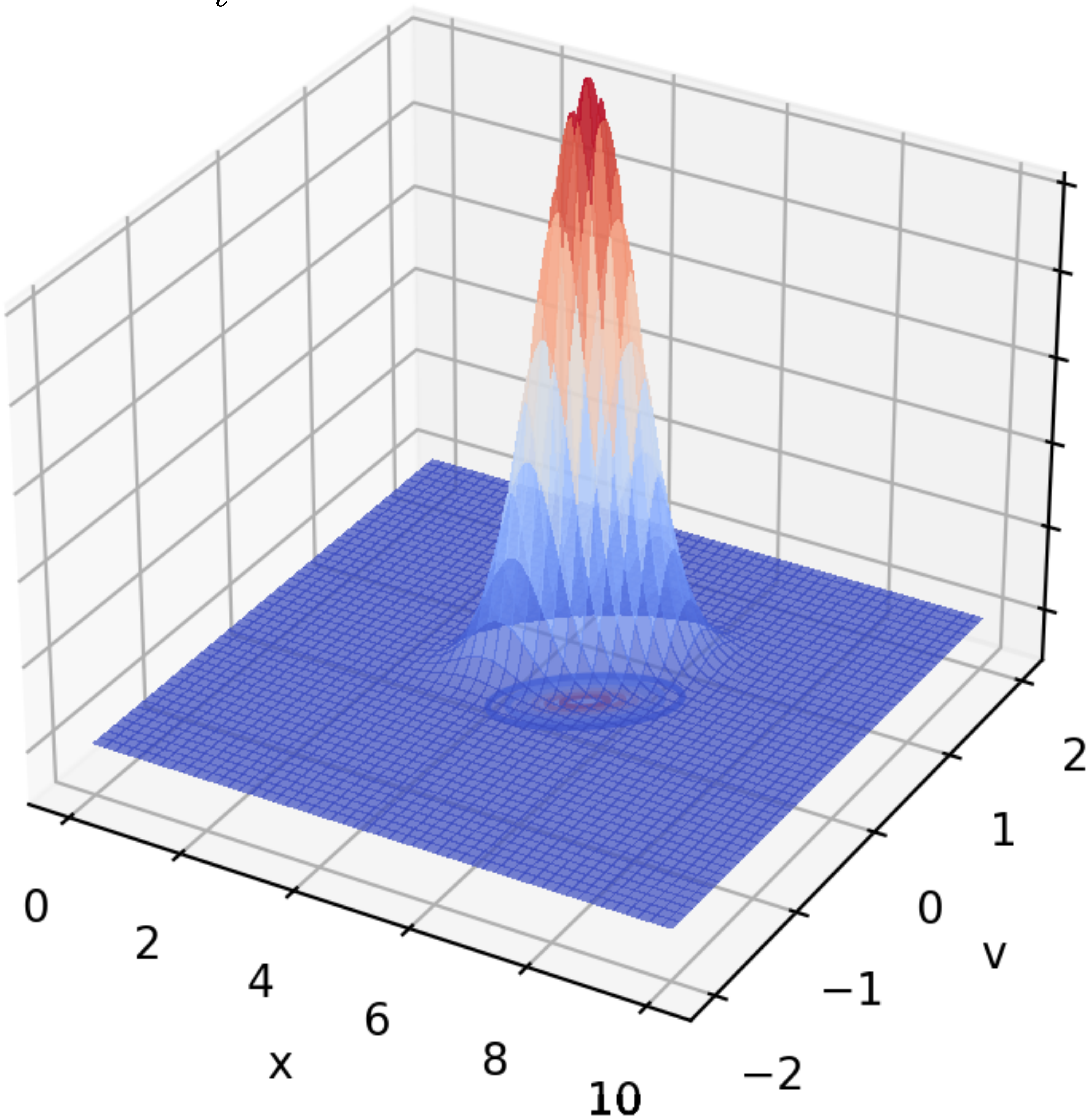
$$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}_{\mathbf{z}_t}\Big(\underbrace{\begin{bmatrix}1 & 0\end{bmatrix}}_{\mathbf{C}_t}\mathbf{x}_{t-1},\ \underbrace{\begin{bmatrix}0.3\end{bmatrix}}_{\mathbf{Q}_t}\Big)$$

2. Prediction step (new action $\mathbf{u}_t$ performed):

$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t\boldsymbol{\mu}_{t-1} + \mathbf{B}_t\mathbf{u}_t$$
$$\overline{\boldsymbol{\Sigma}}_t = \mathbf{A}_t\boldsymbol{\Sigma}_{t-1}\mathbf{A}_t^\top + \mathbf{R}_t$$
$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$$

3. Measurement update (new $\mathbf{z}_t$ received):

$$\mathbf{K}_t = \overline{\boldsymbol{\Sigma}}_t\mathbf{C}_t^\top(\mathbf{C}_t\overline{\boldsymbol{\Sigma}}_t\mathbf{C}_t^\top + \mathbf{Q}_t)^{-1}$$
$$\boldsymbol{\mu}_t = \overline{\boldsymbol{\mu}}_t + \mathbf{K}_t(\mathbf{z}_t - \mathbf{C}_t\overline{\boldsymbol{\mu}}_t)$$
$$\boldsymbol{\Sigma}_t = (\mathbf{I} - \mathbf{K}_t\mathbf{C}_t)\overline{\boldsymbol{\Sigma}}_t$$
$$\mathrm{bel}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$$



KF example: state = (x … position, v …velocity)

$$p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) = \mathcal{N}_{\mathbf{x}_t}\Big(\underbrace{\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}}_{\mathbf{A}_t}\mathbf{x}_{t-1} + \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{B}_t}\mathbf{u}_t,\ \underbrace{\begin{bmatrix}0.01 & 0\\ 0 & 0.01\end{bmatrix}}_{\mathbf{R}_t}\Big)$$

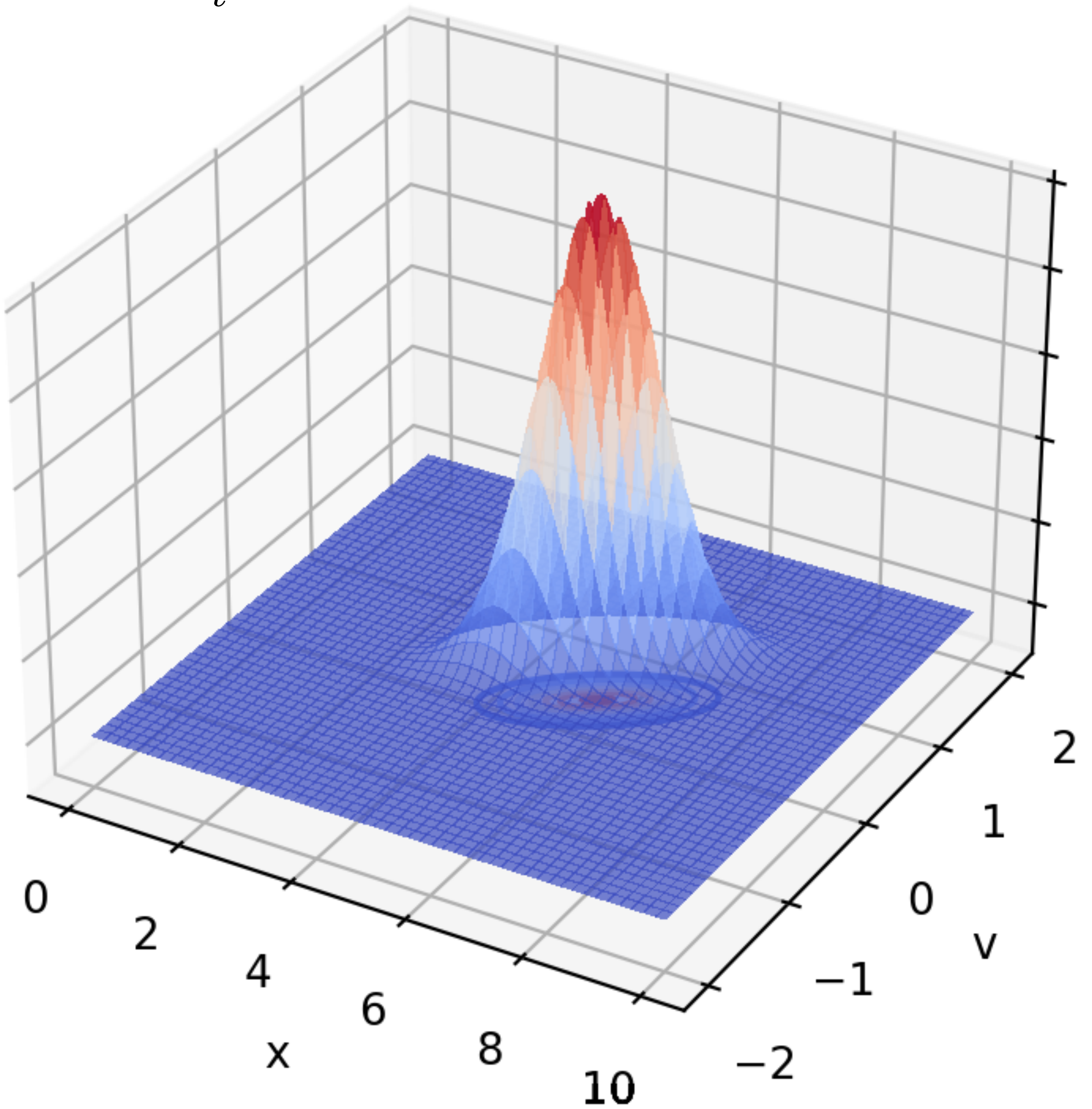
$$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}_{\mathbf{z}_t}\Big(\underbrace{\begin{bmatrix}1 & 0\end{bmatrix}}_{\mathbf{C}_t}\mathbf{x}_{t-1},\ \underbrace{\begin{bmatrix}0.3\end{bmatrix}}_{\mathbf{Q}_t}\Big)$$

2. Prediction step (new action $\mathbf{u}_t$ performed):

$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t\boldsymbol{\mu}_{t-1} + \mathbf{B}_t\mathbf{u}_t$$
$$\overline{\boldsymbol{\Sigma}}_t = \mathbf{A}_t\boldsymbol{\Sigma}_{t-1}\mathbf{A}_t^\top + \mathbf{R}_t$$
$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$$

3. Measurement update (new $\mathbf{z}_t$ received):

$$\mathbf{K}_t = \overline{\boldsymbol{\Sigma}}_t\mathbf{C}_t^\top(\mathbf{C}_t\overline{\boldsymbol{\Sigma}}_t\mathbf{C}_t^\top + \mathbf{Q}_t)^{-1}$$
$$\boldsymbol{\mu}_t = \overline{\boldsymbol{\mu}}_t + \mathbf{K}_t(\mathbf{z}_t - \mathbf{C}_t\overline{\boldsymbol{\mu}}_t)$$
$$\boldsymbol{\Sigma}_t = (\mathbf{I} - \mathbf{K}_t\mathbf{C}_t)\overline{\boldsymbol{\Sigma}}_t$$
$$\mathrm{bel}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$$



KF example: state = (x … position, v …velocity)

$$p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) = \mathcal{N}_{\mathbf{x}_t}\Big(\underbrace{\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}}_{\mathbf{A}_t}\mathbf{x}_{t-1} + \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{B}_t}\mathbf{u}_t,\ \underbrace{\begin{bmatrix}0.01 & 0\\ 0 & 0.01\end{bmatrix}}_{\mathbf{R}_t}\Big)$$

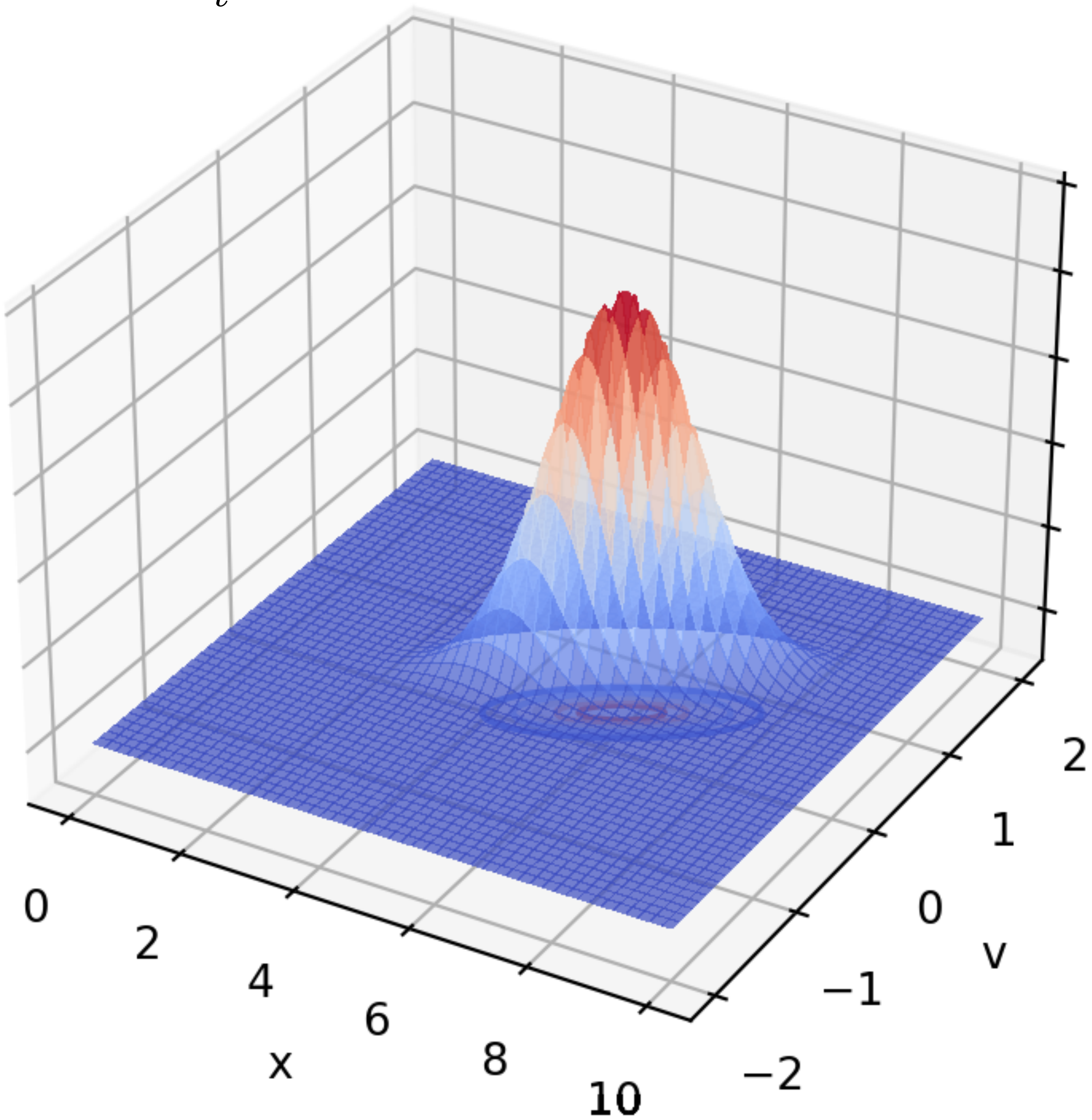
$$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}_{\mathbf{z}_t}\Big(\underbrace{\begin{bmatrix}1 & 0\end{bmatrix}}_{\mathbf{C}_t}\mathbf{x}_{t-1},\ \underbrace{\begin{bmatrix}0.3\end{bmatrix}}_{\mathbf{Q}_t}\Big)$$

2. Prediction step (new action $\mathbf{u}_t$ performed):

$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t\boldsymbol{\mu}_{t-1} + \mathbf{B}_t\mathbf{u}_t$$
$$\overline{\boldsymbol{\Sigma}}_t = \mathbf{A}_t\boldsymbol{\Sigma}_{t-1}\mathbf{A}_t^\top + \mathbf{R}_t$$
$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$$

3. Measurement update (new $\mathbf{z}_t$ received):

$$\mathbf{K}_t = \overline{\boldsymbol{\Sigma}}_t\mathbf{C}_t^\top(\mathbf{C}_t\overline{\boldsymbol{\Sigma}}_t\mathbf{C}_t^\top + \mathbf{Q}_t)^{-1}$$
$$\boldsymbol{\mu}_t = \overline{\boldsymbol{\mu}}_t + \mathbf{K}_t(\mathbf{z}_t - \mathbf{C}_t\overline{\boldsymbol{\mu}}_t)$$
$$\boldsymbol{\Sigma}_t = (\mathbf{I} - \mathbf{K}_t\mathbf{C}_t)\overline{\boldsymbol{\Sigma}}_t$$
$$\mathrm{bel}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$$



Summary Kalman Filter

- Kalman filter is **optimal observer** of the current state for **linear** systems under **Gaussian** noise for **complete** states
- It can also estimate previous states via Kalman smoothing
- Kalman filter is Bayes filter where measurement and transition probabilities are linear-gaussians.
- It nicely scales to higher dimension but the linearity and gaussianity yields significant limitations
    - Example 18-dimensional state space
        - Dicrete bel: Each dimension 10 discrete values => 10^18 parameters
        - KF bel: Continuous Gaussian representation ==> 18^2+18=342 params
- Extended Kalman filter removes the linearity limitation but loses the optimality