Relative motion from <u>unknown</u> correspondences

Iterative Closest Point SLAM

Absolute orientation problem in SE(2)

$$\mathbf{z}^{v} = \arg\min_{\mathbf{t},\theta} \sum_{i} \left\| \mathbf{R}_{\theta} \mathbf{p} + \mathbf{t} - \mathbf{q} \right\|^{2} = \arg\min_{\mathbf{t},\theta} \sum_{i} \left\| \mathbf{R}_{\theta} \mathbf{p}_{i}' - \mathbf{q}_{i}' \right\|^{2} + \left\| \mathbf{R}_{\theta} \tilde{\mathbf{p}} + \mathbf{t} - \tilde{\mathbf{q}} \right\|^{2}$$
Substitution:
$$\mathbf{p}_{i}' = \mathbf{p}_{i} - \frac{1}{N} \sum_{i} \mathbf{p}_{i}, \quad \mathbf{q}_{i}' = \mathbf{q}_{i} - \frac{1}{N} \sum_{i} \mathbf{q}_{i}$$
Can be always zero by appropriate choice of \mathbf{t}
Depends only on θ

Solution:
$$\mathbf{H} = \sum_{i} \mathbf{p}'_{i} \mathbf{q}'_{i}^{\mathsf{T}} \dots$$
 covariance matrix

$$\theta^* = \arg\min_{\theta} \sum_{i} \|\mathbf{R}_{\theta} \mathbf{p}'_i - \mathbf{q}'_i\|^2 = \arctan\left(\frac{H_{xy} - H_{yx}}{H_{xx} + H_{yy}}\right)$$

$$\mathbf{t}^* = \arg\min_{\mathbf{t}} \|\mathbf{R}_{\theta^*} \tilde{\mathbf{p}} + \mathbf{t} - \tilde{\mathbf{q}}\|^2 = \tilde{\mathbf{q}} - \mathbf{R}_{\theta^*} \tilde{\mathbf{p}}$$

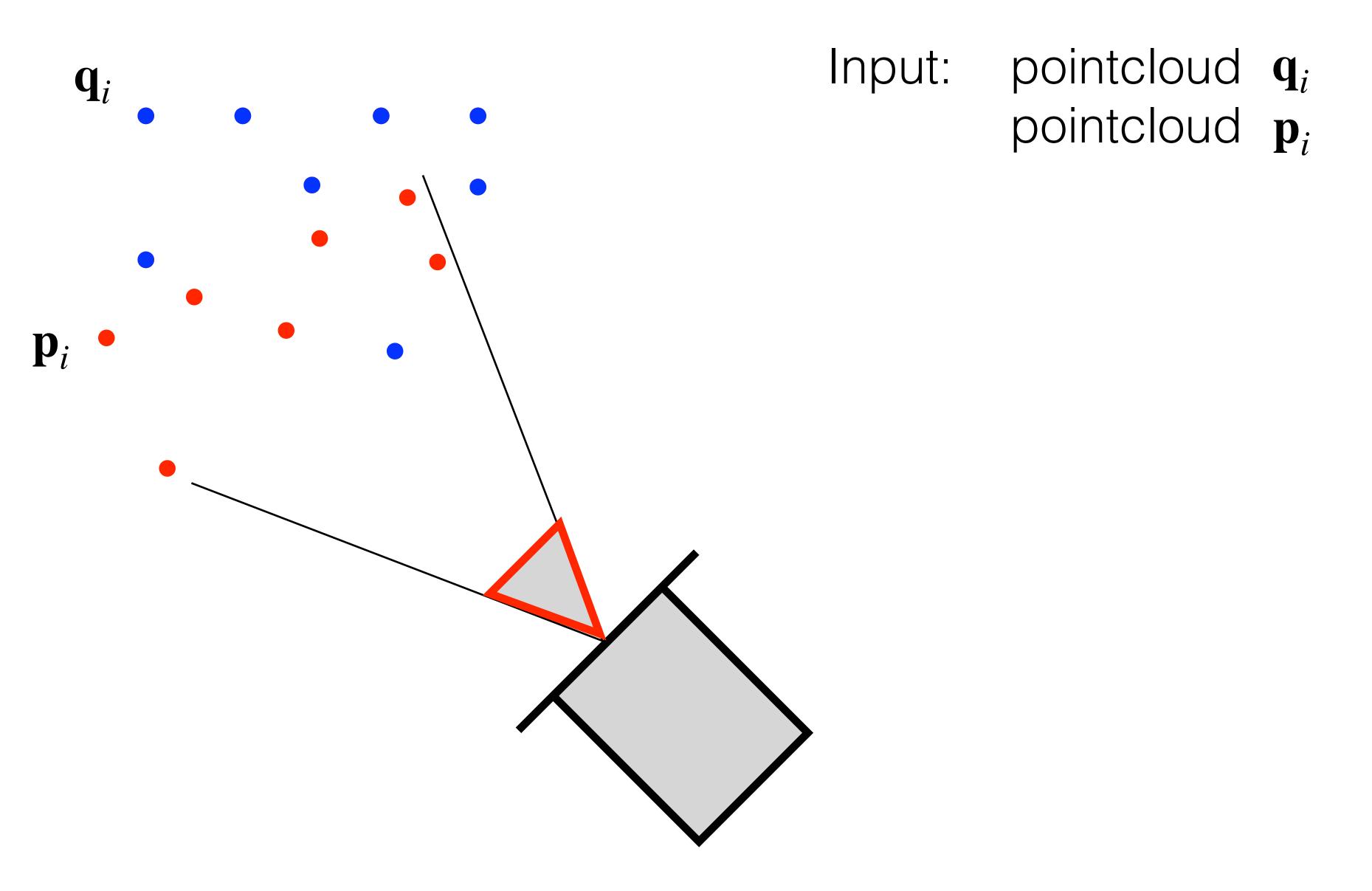
Absolute orientation problem in SE(3)

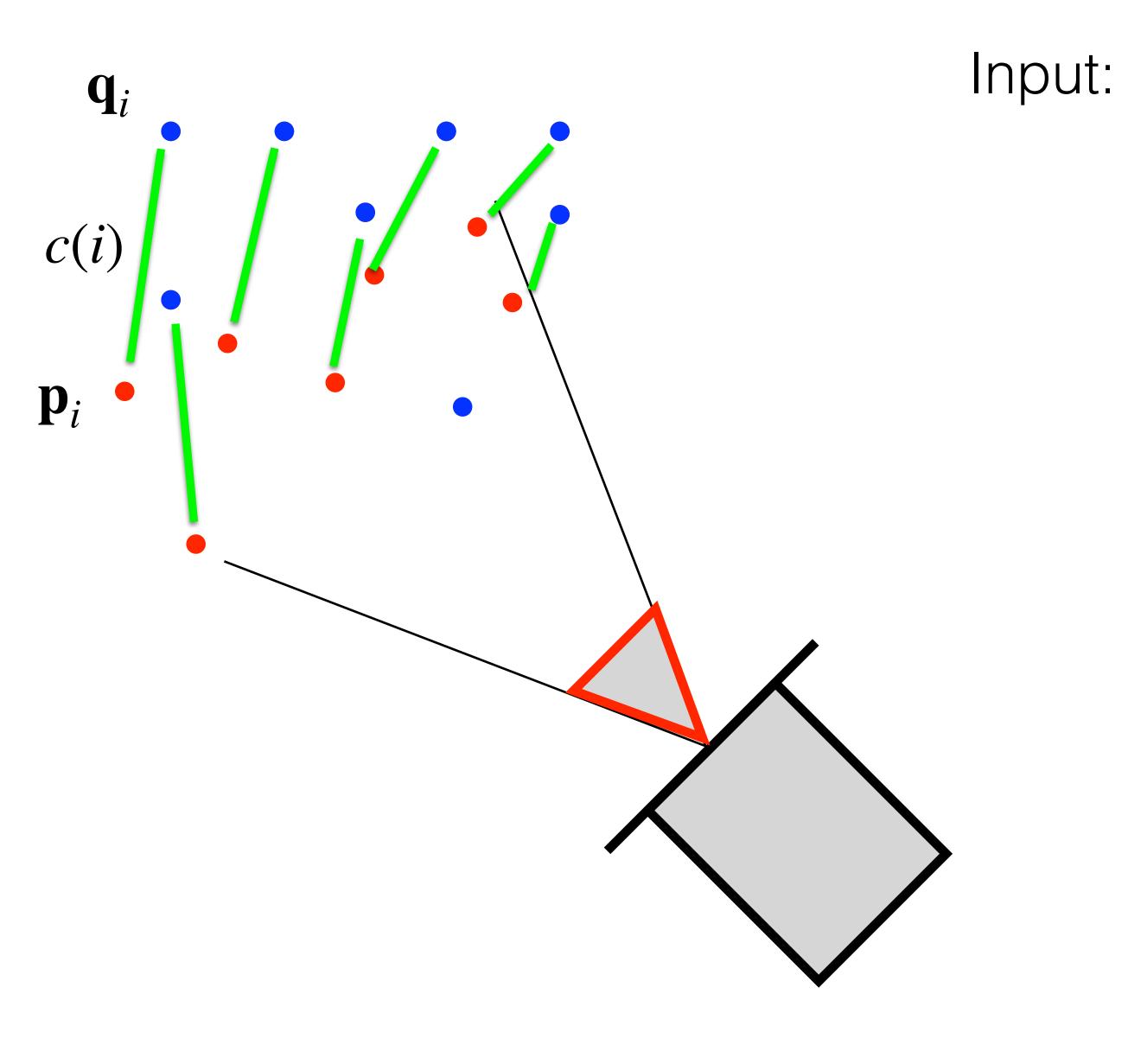
$$\mathbf{z}^{v} = \arg\min_{\mathbf{t},\mathbf{R}} \sum_{i} \| \mathbf{R}\mathbf{p} + \mathbf{t} - \mathbf{q} \|^{2} = \arg\min_{\mathbf{t},\mathbf{R}} \sum_{i} \| \mathbf{R}\mathbf{p}_{i}' - \mathbf{q}_{i}' \|^{2} + \| \mathbf{R}\tilde{\mathbf{p}} + \mathbf{t} - \tilde{\mathbf{q}} \|^{2}$$
Substitution:
$$\mathbf{p}_{i}' = \mathbf{p}_{i} - \frac{1}{N} \sum_{i} \mathbf{p}_{i}, \quad \mathbf{q}_{i}' = \mathbf{q}_{i} - \frac{1}{N} \sum_{i} \mathbf{q}_{i}$$
Can be always zero by appropriate choice of \mathbf{t}
Depends only on \mathbf{R}

Solution:
$$\mathbf{H} = \sum_{i} \mathbf{p}'_{i} \mathbf{q}'_{i}^{\mathsf{T}} \dots$$
 covariance matrix with SVD decomposition $\mathbf{H} = \mathbf{U} \mathbf{S} \mathbf{V}^{\mathsf{T}}$

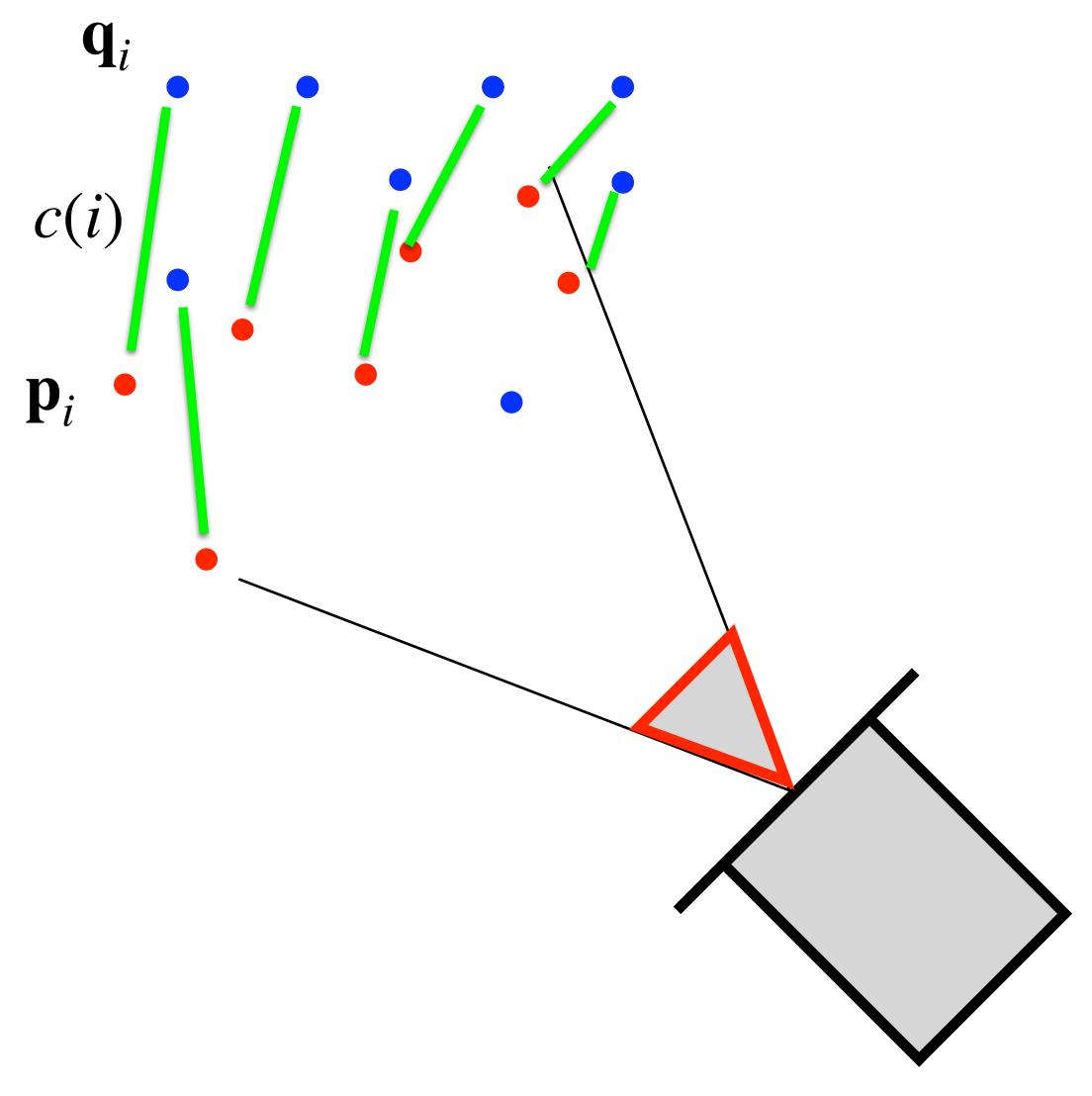
$$\mathbf{R}^* = \arg\min_{\mathbf{R}} \sum_{i} \|\mathbf{R}\mathbf{p}_i' - \mathbf{q}_i'\|^2 = \mathbf{V}\mathbf{U}^\top$$

$$\mathbf{t}^* = \arg\min_{\mathbf{t}} \|\mathbf{R}^* \tilde{\mathbf{p}} + \mathbf{t} - \tilde{\mathbf{q}}\|^2 = \tilde{\mathbf{q}} - \mathbf{R}^* \tilde{\mathbf{p}}$$



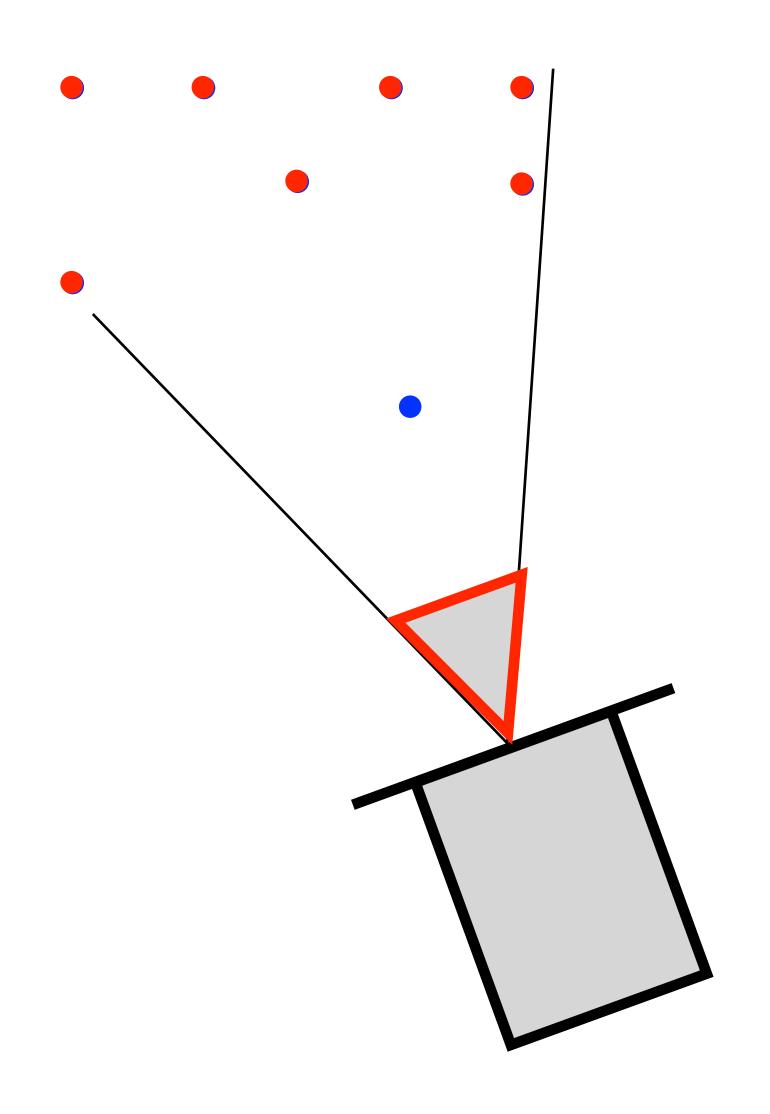


pointcloud \mathbf{q}_i pointcloud \mathbf{p}_i correspondences c(i)



Input: pointcloud \mathbf{q}_i pointcloud \mathbf{p}_i correspondences c(i)

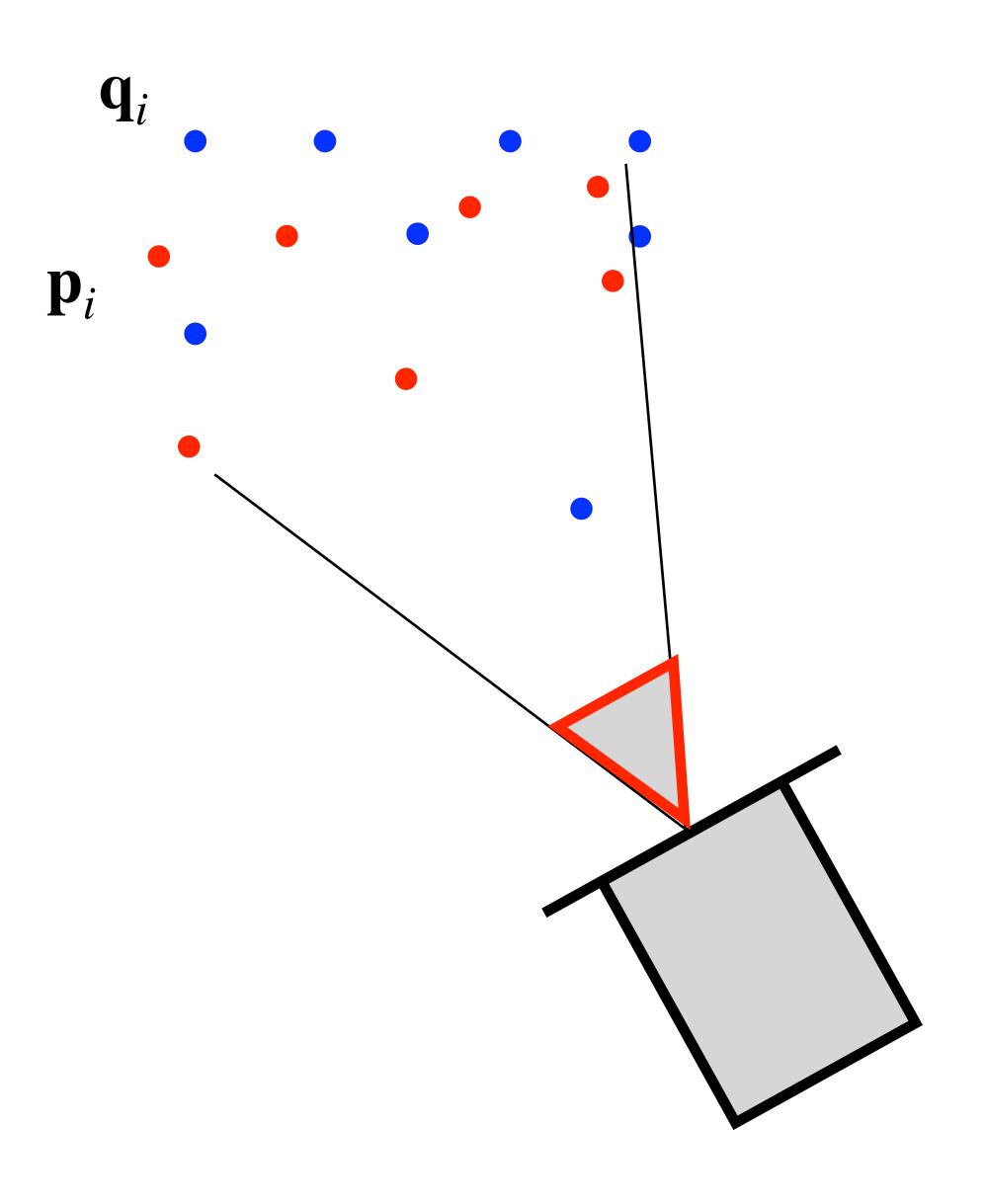
$$\mathbf{R}^{\star}, \mathbf{t}^{\star}, = \underset{\mathbf{R} \in SO(3), \mathbf{t}}{\operatorname{arg min}} \sum_{i} \| \mathbf{R} \mathbf{p}_{i} + \mathbf{t} - \mathbf{q}_{c(i)} \|^{2}$$



Input: pointcloud
$$\mathbf{q}_i$$
 pointcloud \mathbf{p}_i correspondences $c(i)$

1. Solve absolute orientation:

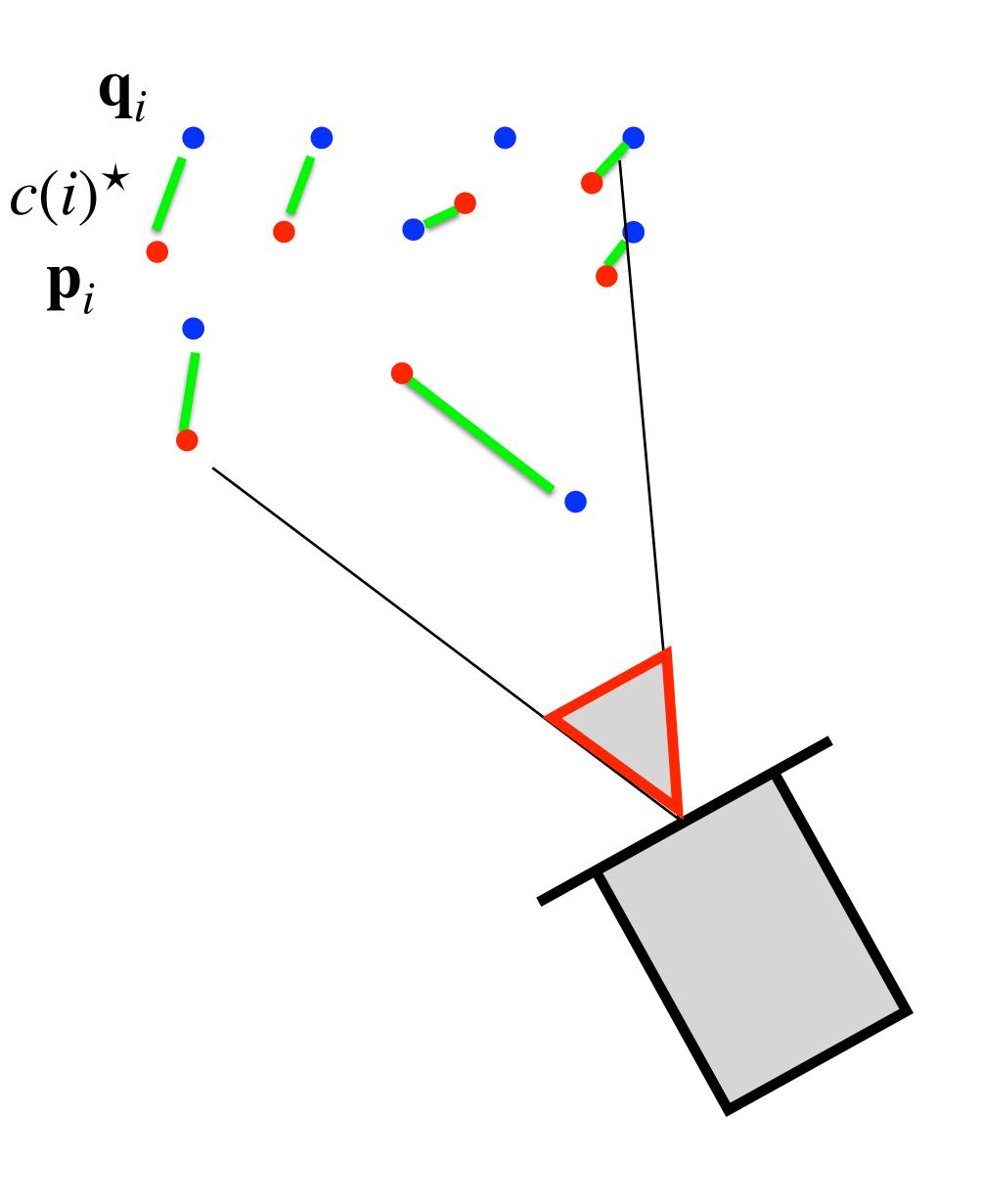
$$\mathbf{R}^{\star}, \mathbf{t}^{\star}, = \underset{\mathbf{R} \in SO(3), \mathbf{t}}{\operatorname{arg min}} \sum_{i} \| \mathbf{R} \mathbf{p}_{i} + \mathbf{t} - \mathbf{q}_{c(i)} \|^{2}$$



Input: pointcloud
$$\mathbf{q}_i$$
 pointcloud \mathbf{p}_i correspondences $c(i)$

- 1. Initialize $\mathbf{R}^* = \mathbf{R}_0$, $\mathbf{t}^* = \mathbf{t}_0$
- 2. Solve nearest neighbour:

$$c(i)^* = \underset{c(i)}{\operatorname{arg\,min}} \sum_{i} \left\| \mathbf{R}^* \mathbf{p}_i + \mathbf{t}^* - \mathbf{q}_{c(i)} \right\|^2$$

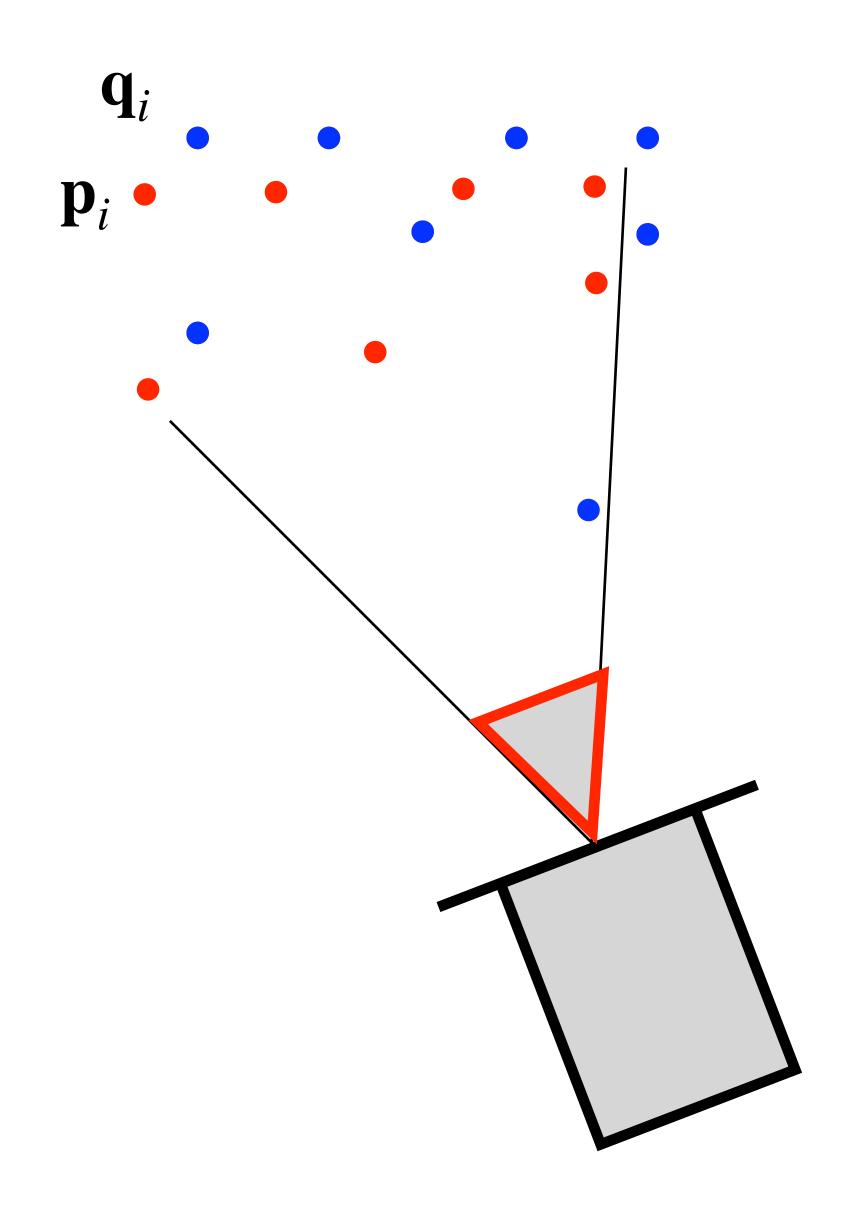


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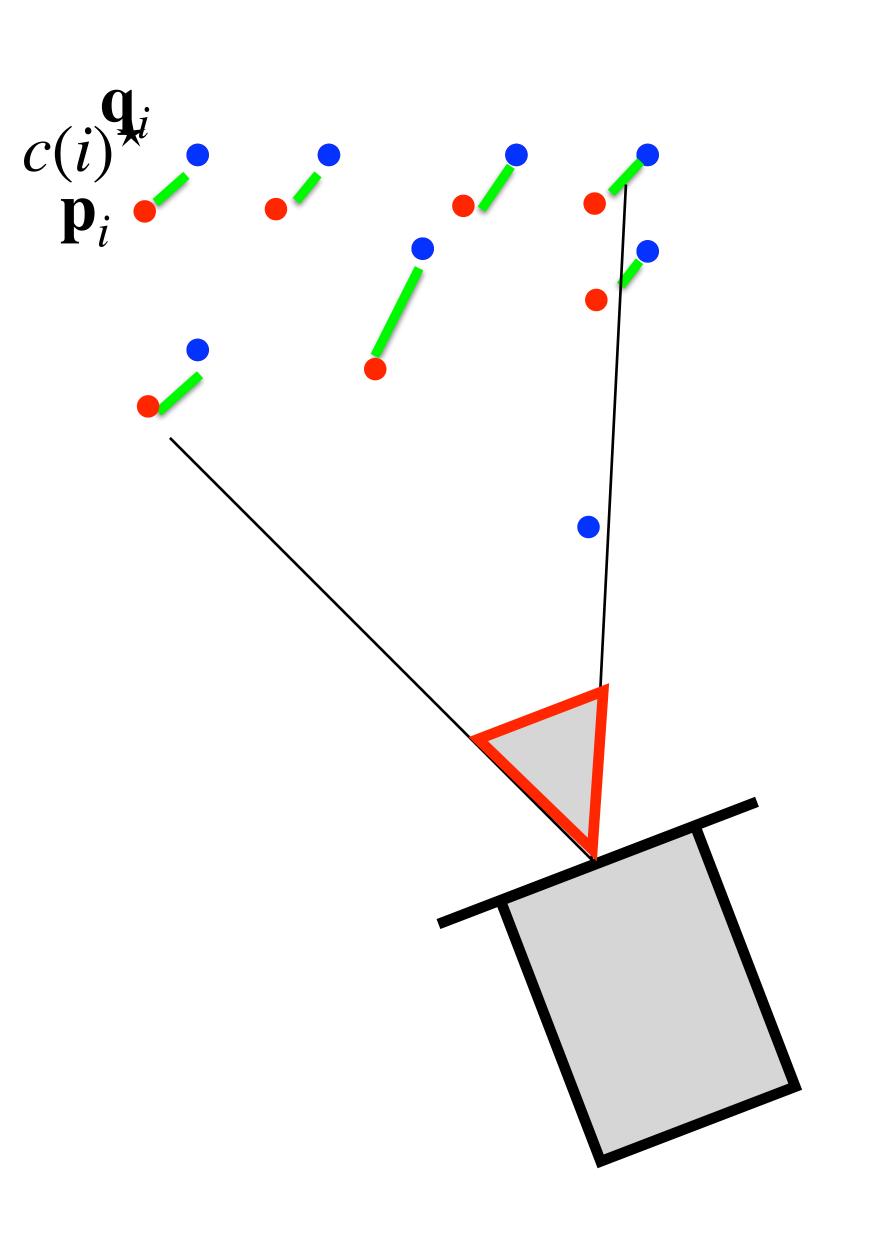


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 pointcloud \mathbf{p}_i correspondences $c(i)$

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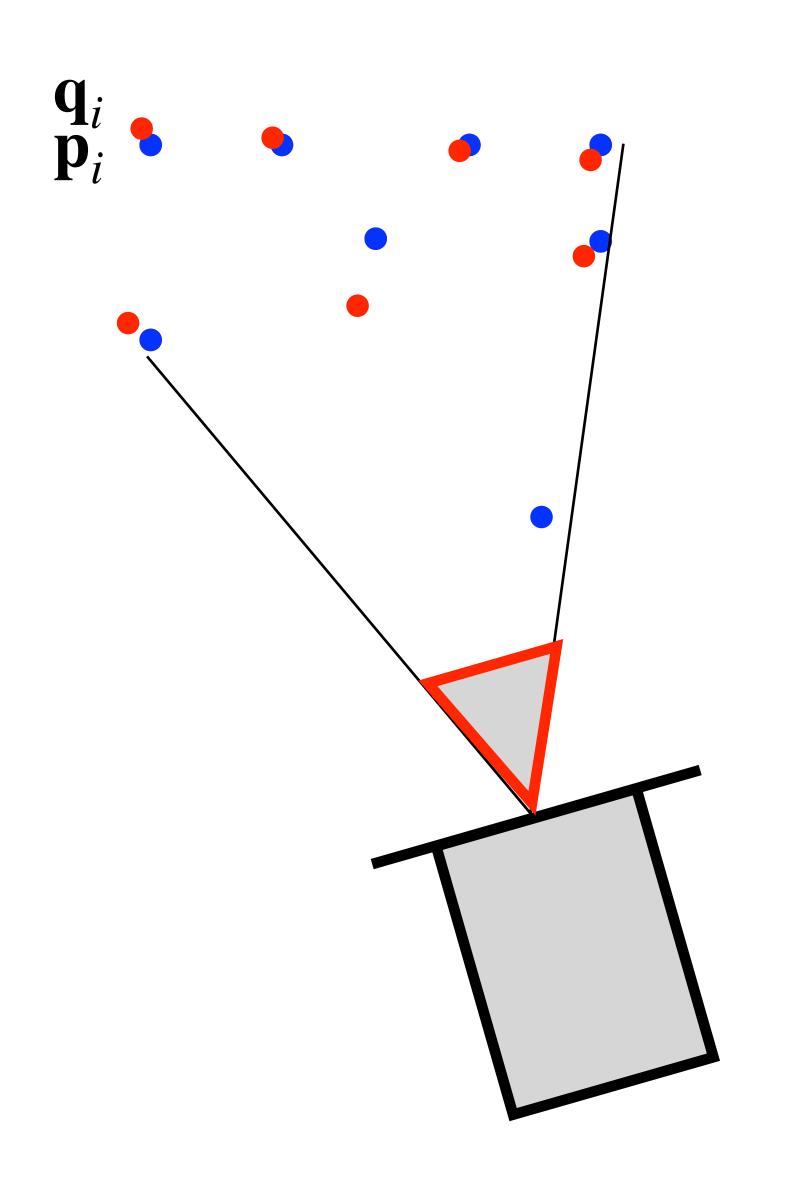
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$$\mathbf{R}^{\star}, \mathbf{t}^{\star}, = \underset{\mathbf{R} \in SO(3), \mathbf{t}}{\operatorname{arg min}} \sum_{i} \| \mathbf{R} \mathbf{p}_{i} + \mathbf{t} - \mathbf{q}_{c(i)^{\star}} \|^{2}$$



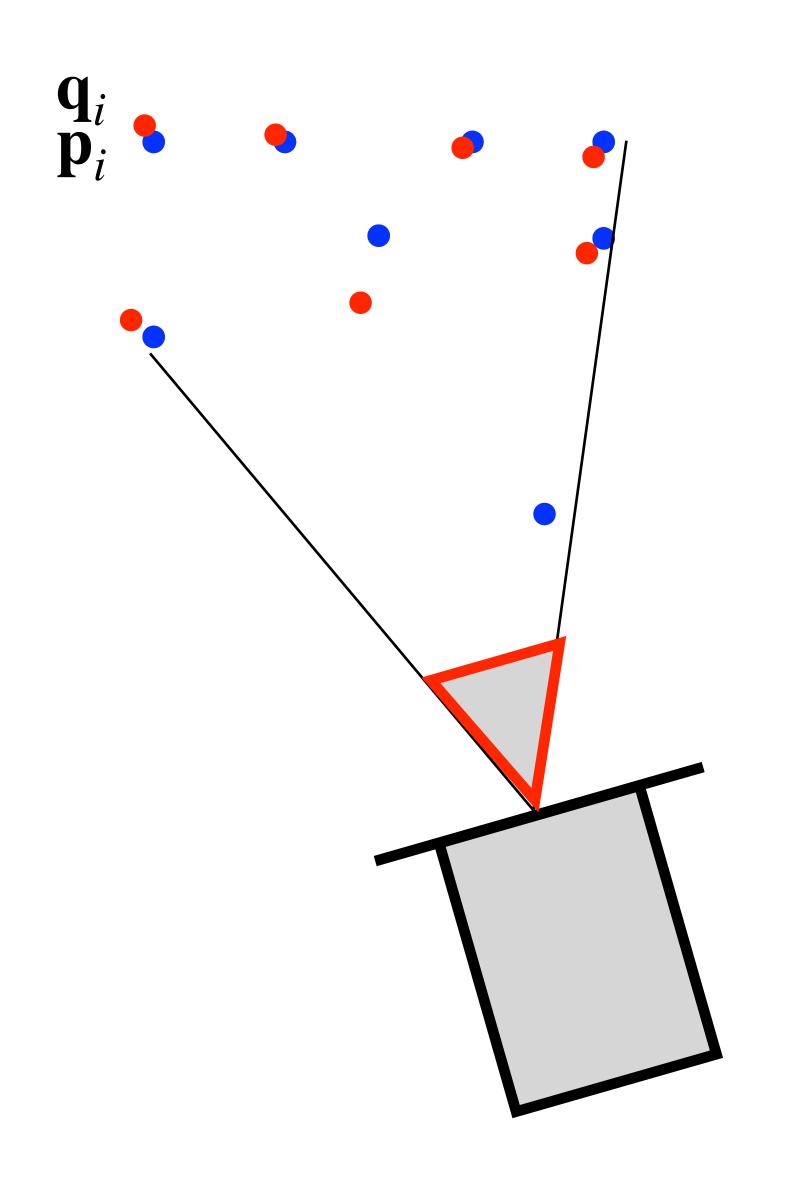
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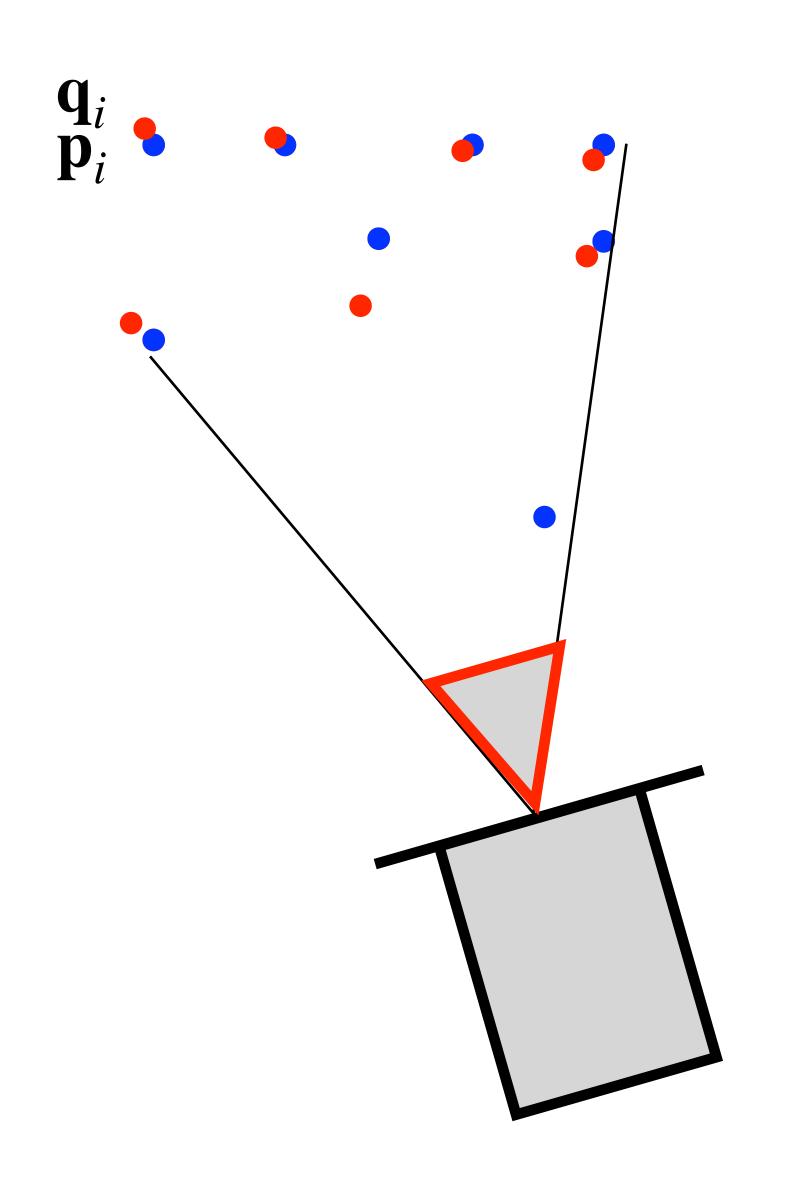
Input: pointcloud
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 pointcloud \mathbf{p}_i correspondences $c(i)$

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3. Solve absolute orientation:

$$\mathbf{R}^{\star}, \mathbf{t}^{\star}, = \underset{\mathbf{R} \in SO(3), \mathbf{t}}{\operatorname{arg min}} \sum_{i} \| \mathbf{R} \mathbf{p}_{i} + \mathbf{t} - \mathbf{q}_{c(i)^{\star}} \|^{2}$$



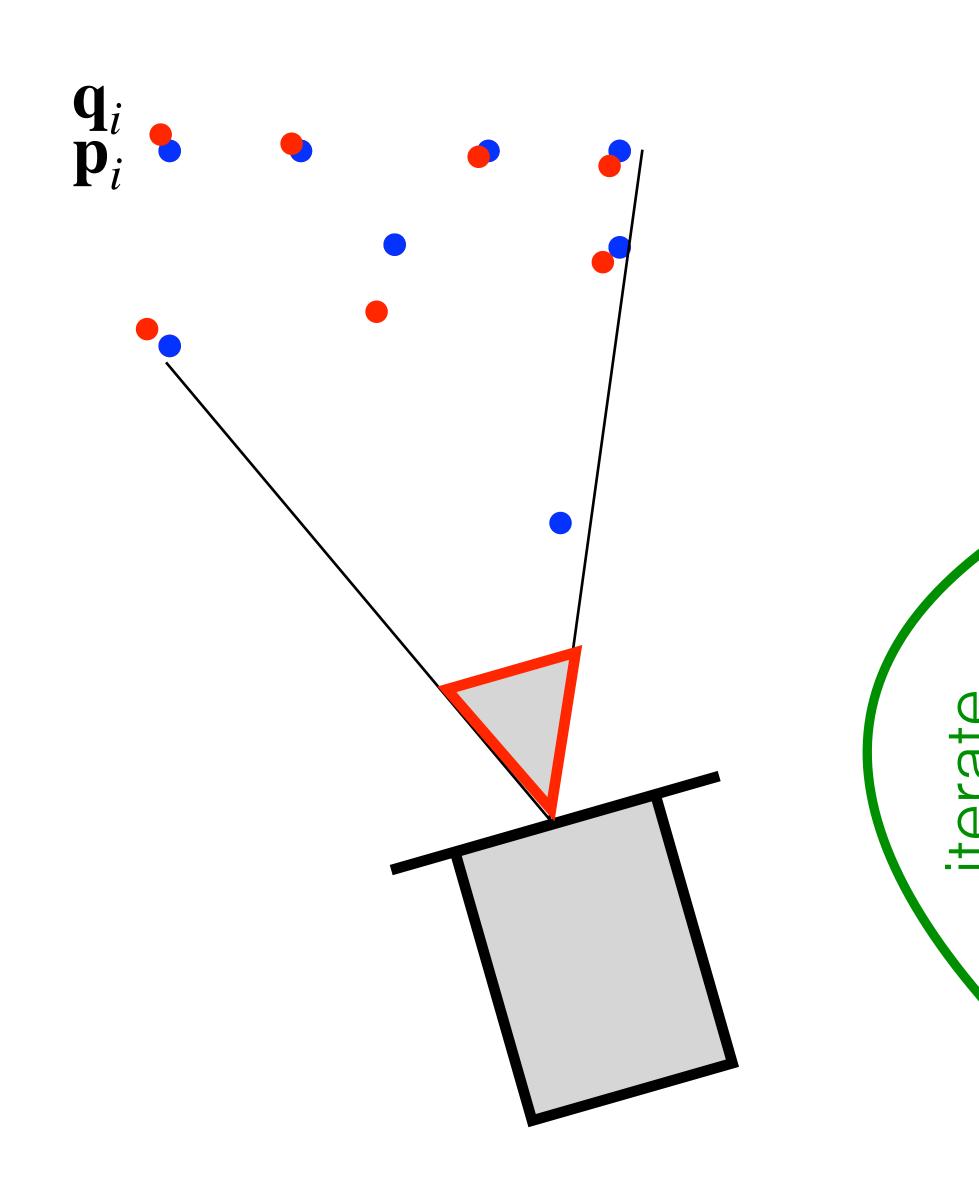
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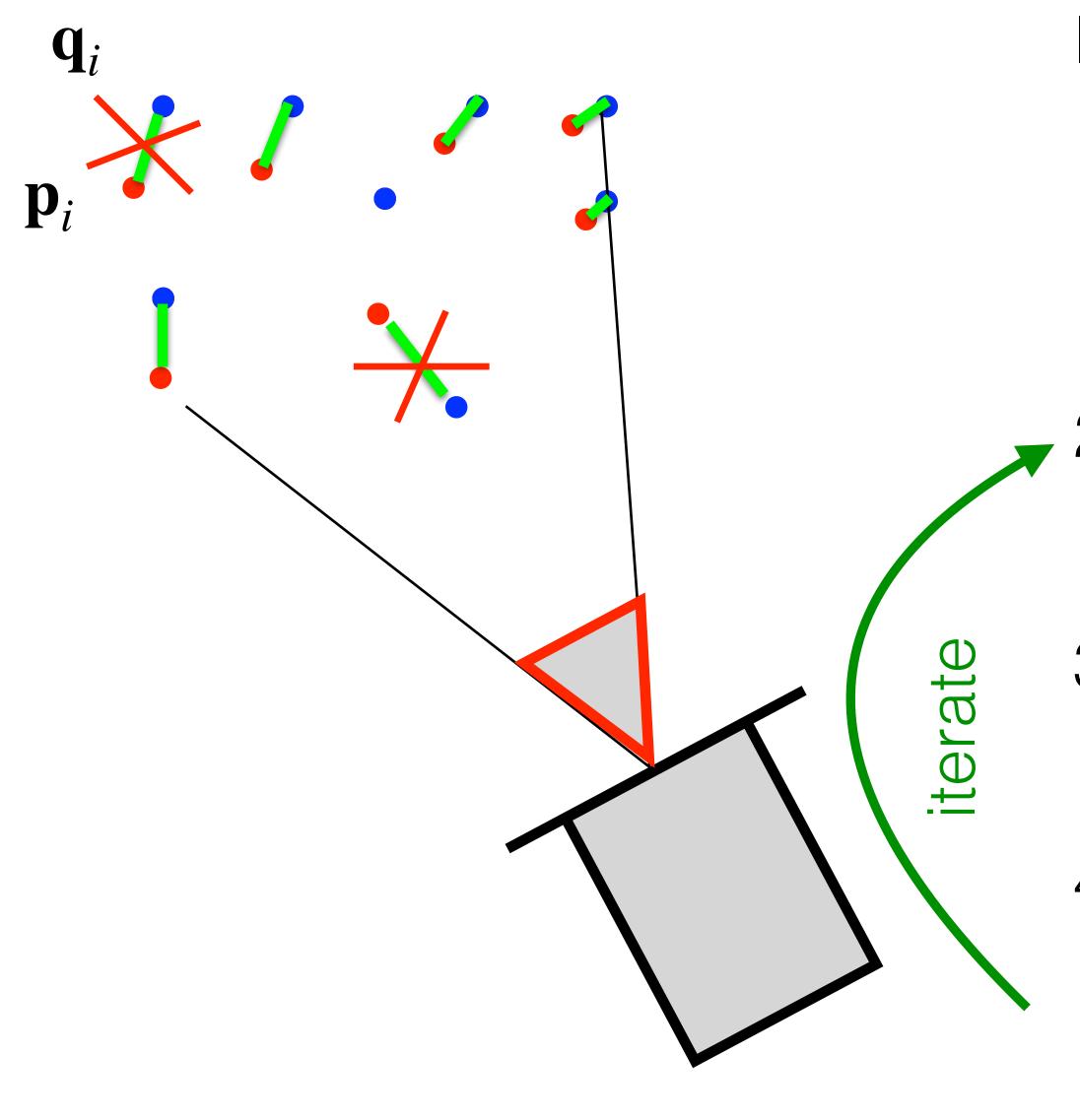
$$c(i)^* = \underset{c(i)}{\operatorname{arg\,min}} \sum_{i} \left\| \mathbf{R}^* \mathbf{p}_i + \mathbf{t}^* - \mathbf{q}_{c(i)} \right\|^2$$

3. Outlier rejection by median thresholding:

if
$$\|\mathbf{R}^*\mathbf{p}_i + \mathbf{t}^* - \mathbf{q}_{c(i)^*}\|^2 \ge \theta$$
 then $c(i)^* = \mathbf{m}$

4. Solve absolute orientation:

$$\mathbf{R}^{\star}, \mathbf{t}^{\star}, = \underset{\mathbf{R} \in SO(3), \mathbf{t}}{\operatorname{arg min}} \sum_{i} \left\| \mathbf{R} \mathbf{p}_{i} + \mathbf{t} - \mathbf{q}_{c(i)^{\star}} \right\|^{2}$$



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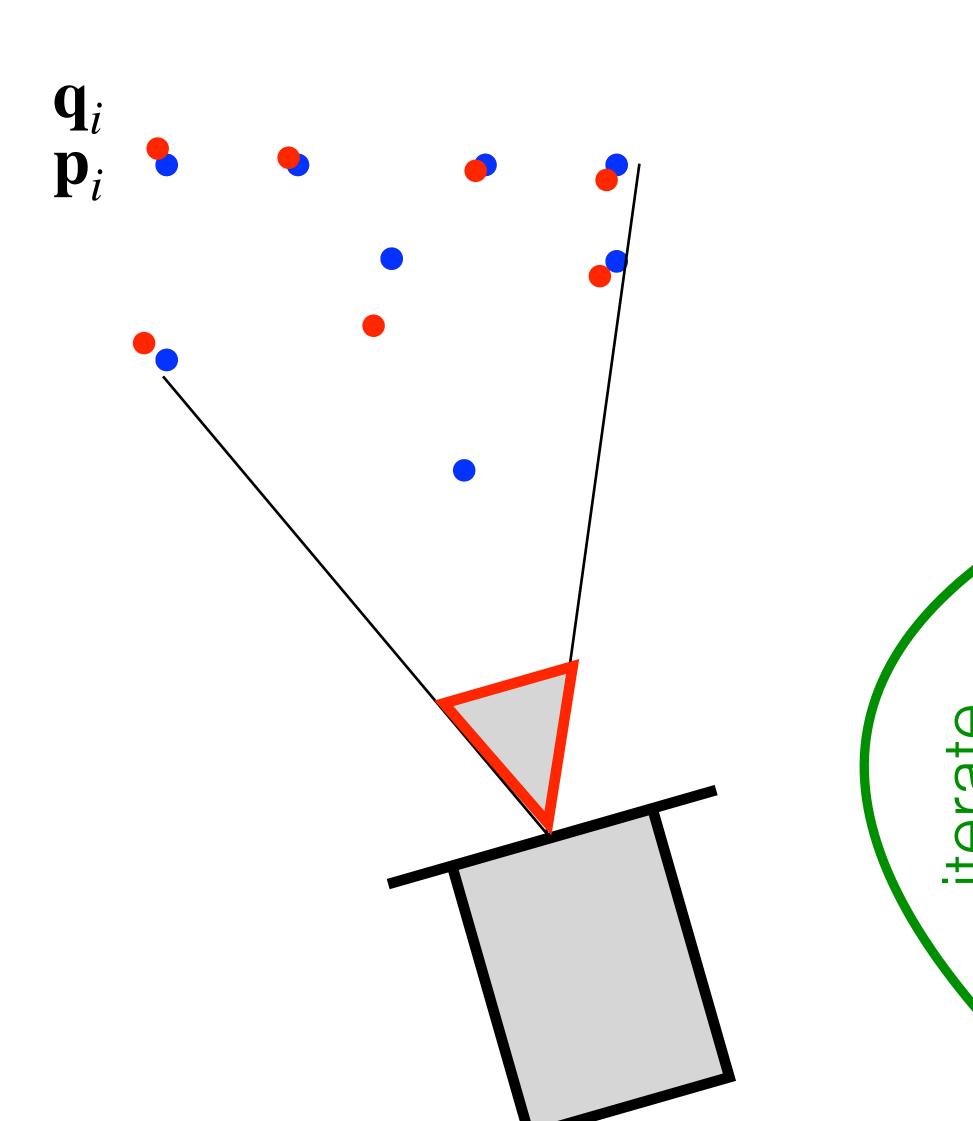
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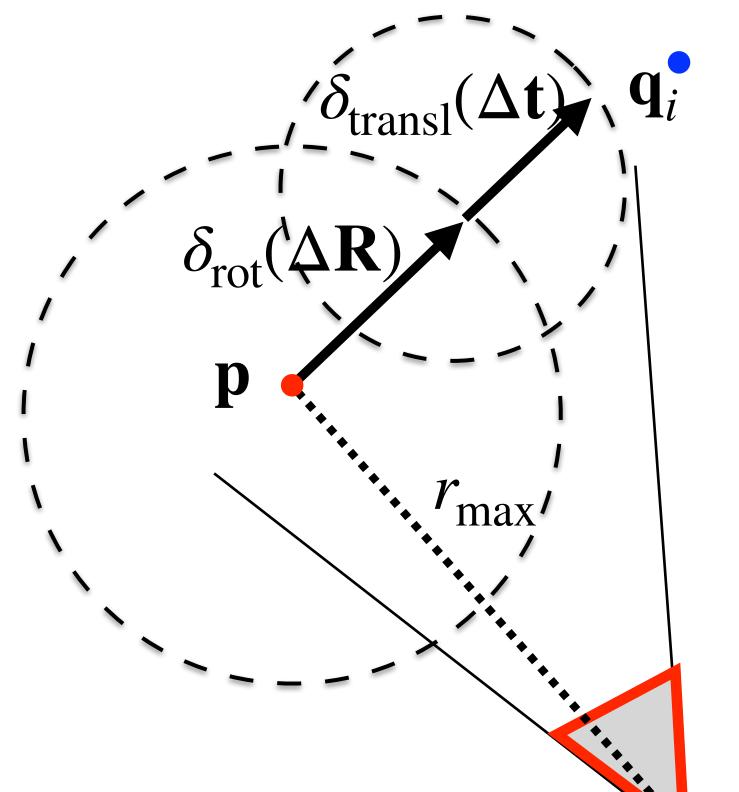
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KISS ICP [2023]

Input: pointcloud \mathbf{q}_i pointcloud \mathbf{p}_i correspondences c(i)

- 1. Initialize $\mathbf{R}^* = \mathbf{R}_0$, $\mathbf{t}^* = \mathbf{t}_0$
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$$c(i)^* = \underset{c(i)}{\operatorname{arg\,min}} \sum_{i} \left\| \mathbf{R}^* \mathbf{p}_i + \mathbf{t}^* - \mathbf{q}_{c(i)} \right\|^2$$

3. Outlier rejection by median thresholding:

if
$$\|\mathbf{R}^*\mathbf{p}_i + \mathbf{t}^* - \mathbf{q}_{c(i)^*}\|^2 \ge \theta$$
 then $c(i)^* = \mathbf{m}$

Max $\Delta \mathbf{R}$, $\Delta \mathbf{t}$ deviations:

$$\delta_{\text{rot}}(\Delta \mathbf{R}) = 2 \ r_{\text{max}} \sin \left(\frac{1}{2} \arccos \left(\frac{\text{tr}(\Delta \mathbf{R}) - 1}{2} \right) \right)$$

$$\delta_{\text{transl}}(\Delta \mathbf{t}) = \|\Delta \mathbf{t}\|$$

UB on outlier rejection threshold:

$$\theta = \delta_{\text{rot}}(\Delta \mathbf{R}) + \delta_{\text{transl}}(\Delta \mathbf{t})$$

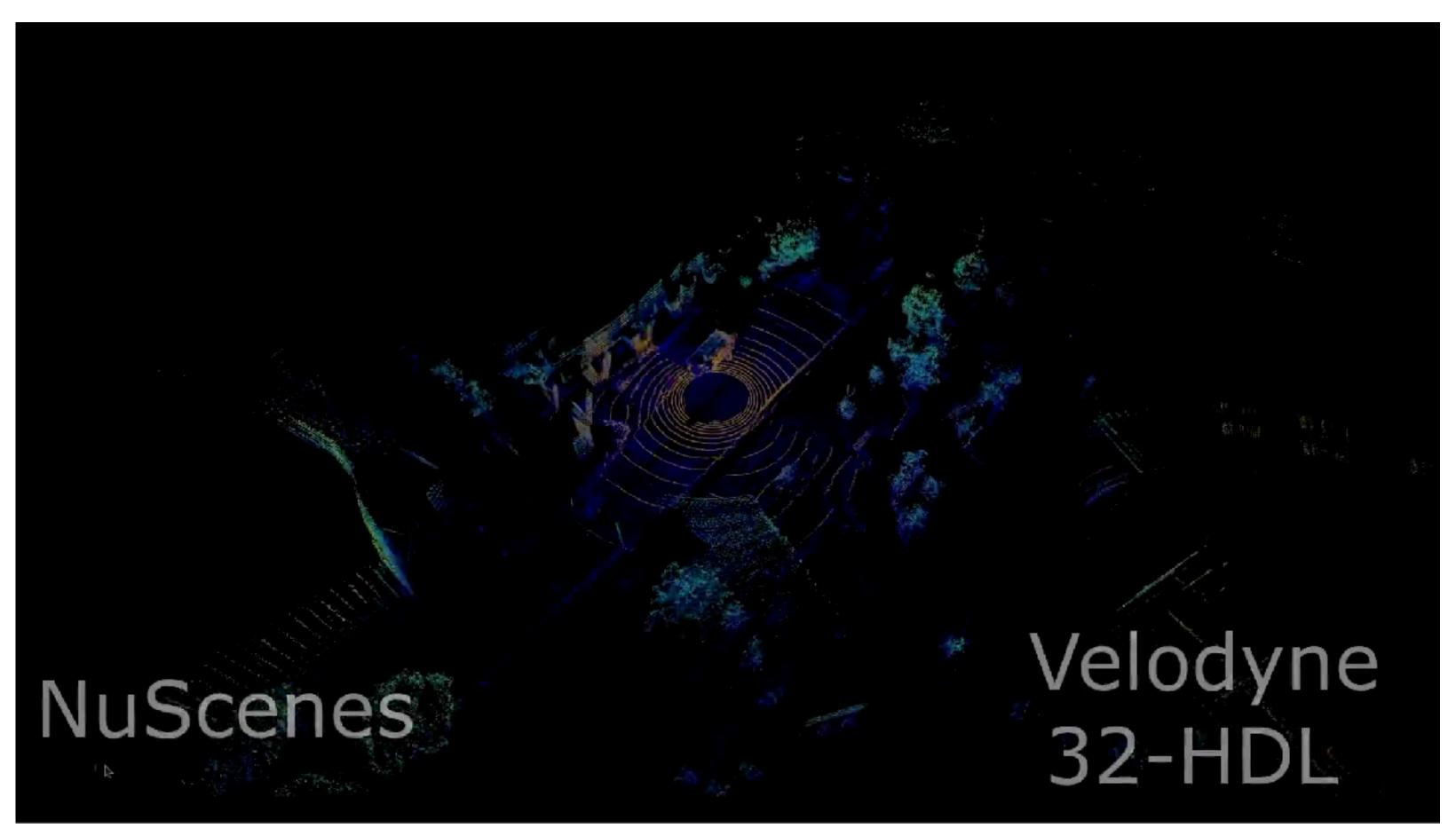
Solve absolute orientation:

$$\mathbf{R}^{\star}, \mathbf{t}^{\star}, = \underset{\mathbf{R} \in SO(3), \mathbf{t}}{\operatorname{arg min}} \sum_{i} \rho \left(\left\| \mathbf{R} \mathbf{p}_{i} + \mathbf{t} - \mathbf{q}_{c(i)^{\star}} \right\|^{2} \right)$$

Output: $\mathbf{R}^*, \mathbf{t}^* \Rightarrow \mathbf{z}_t^v$ https://arxiv.org/pdf/2209.15397.pdf

Pose from unknown correspondences with outlier rejection KISS ICP [2023]

- Good generalization (same params for various car/drone/segway/handheld lidars)
- ROS compatible code: https://github.com/PRBonn/kiss-icp
- Second best opensource approach on autonomous driving dataset challenge (Kitti)



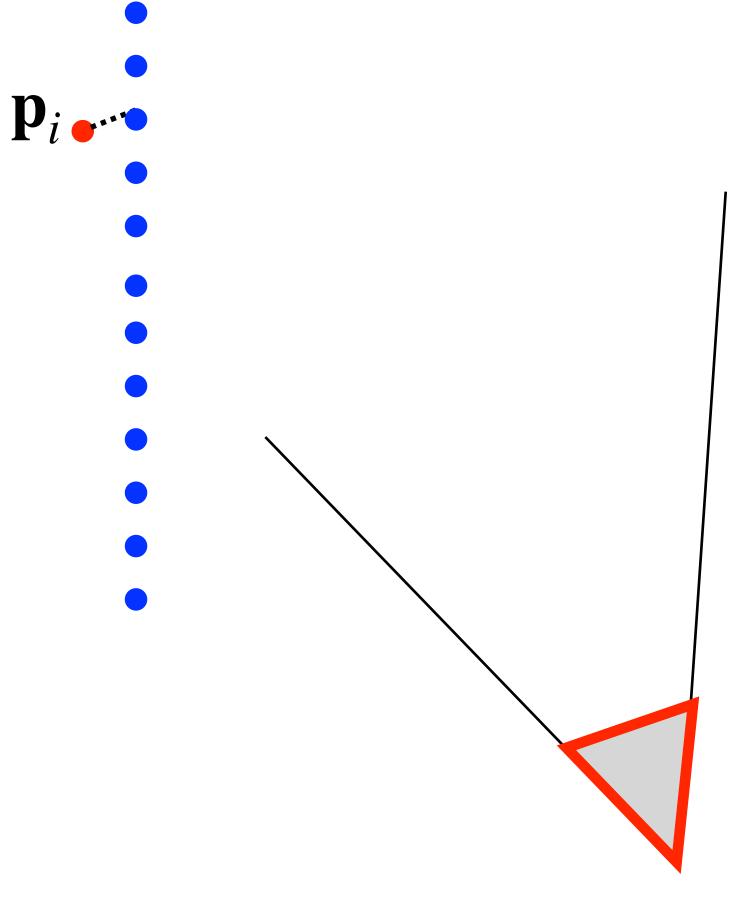
Why do they show results on autonomous car dataset?

Converges to a local minima with unknown region of convergence
 => good initialization needed

- Converges to a local minima with unknown region of convergence
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- Fails in degenerate environments
 (e.g. forward motion in long hallways, or rotation in circular room)
 => other models (point2plane) and sensors (camera) often introduced.

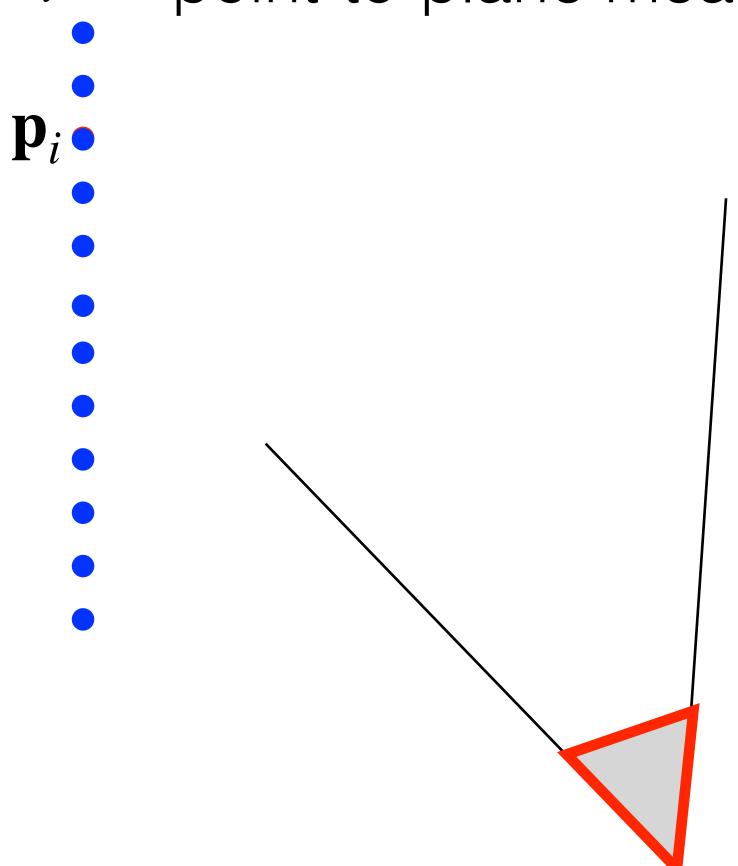
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 \mathbf{q}_i point-to-plane measurement probability model

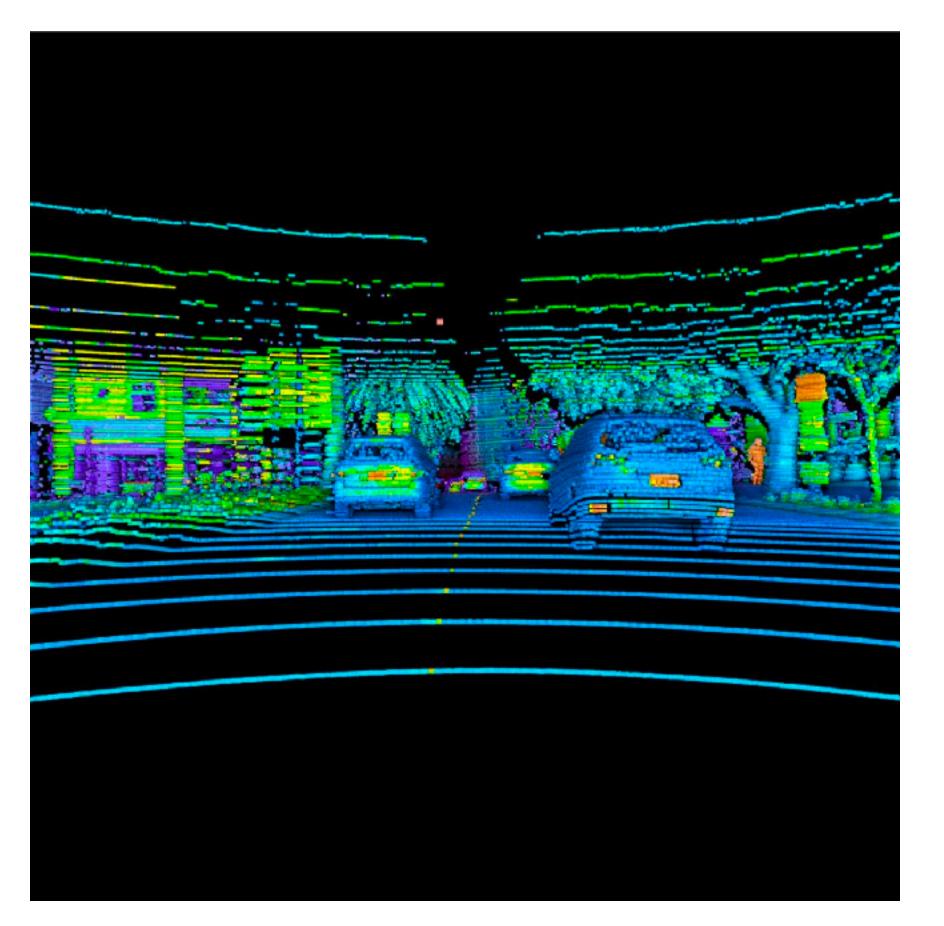


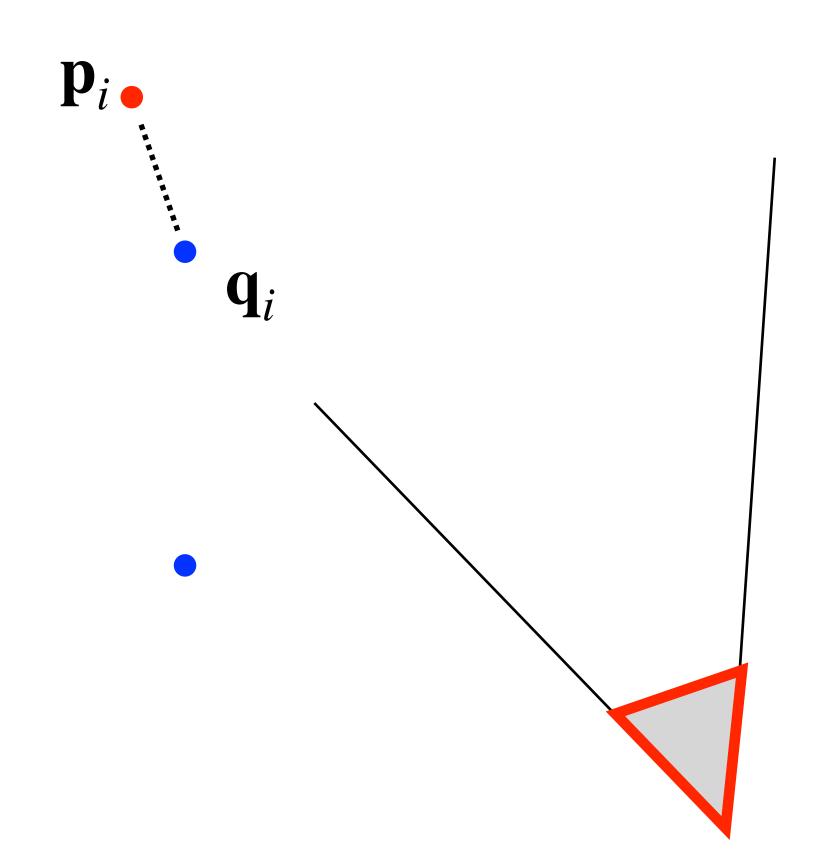
$$\mathbf{R}^{\star}, \mathbf{t}^{\star}, = \underset{\mathbf{R} \in SO(3), \mathbf{t}}{\operatorname{arg min}} \sum_{i} \| \mathbf{R} \mathbf{p}_{i} + \mathbf{t} - \mathbf{q}_{i} \|^{2}$$



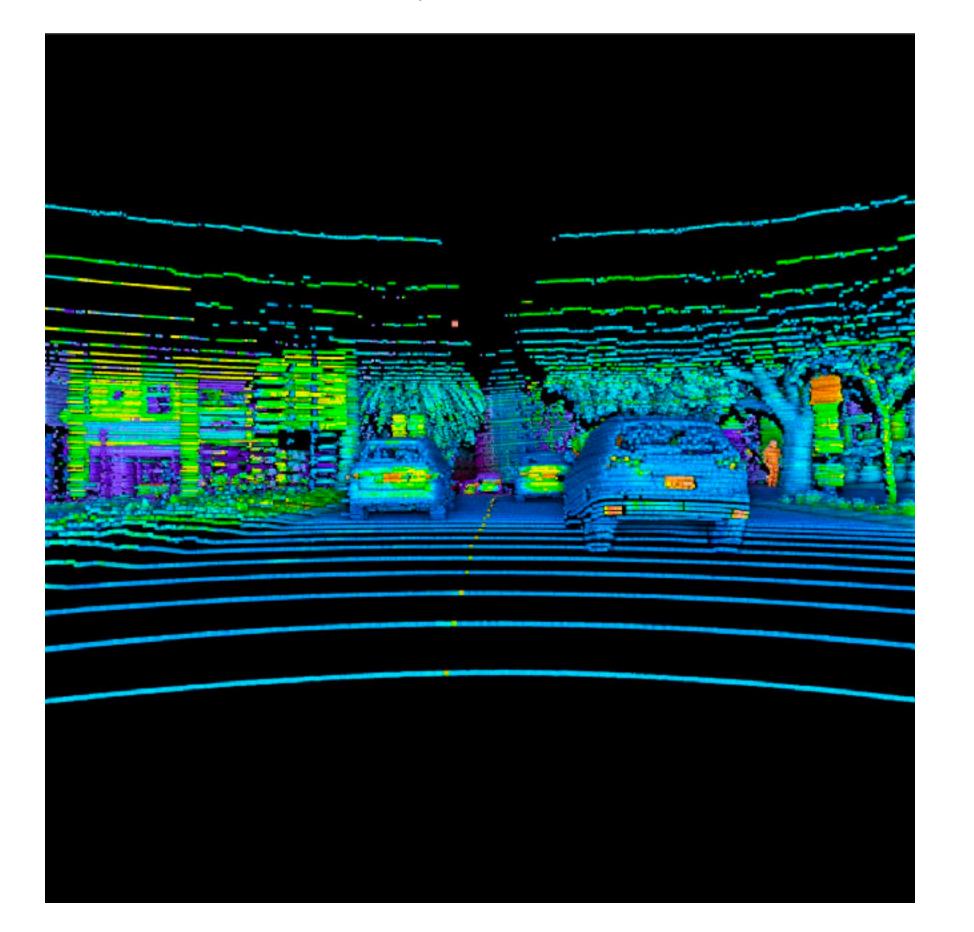


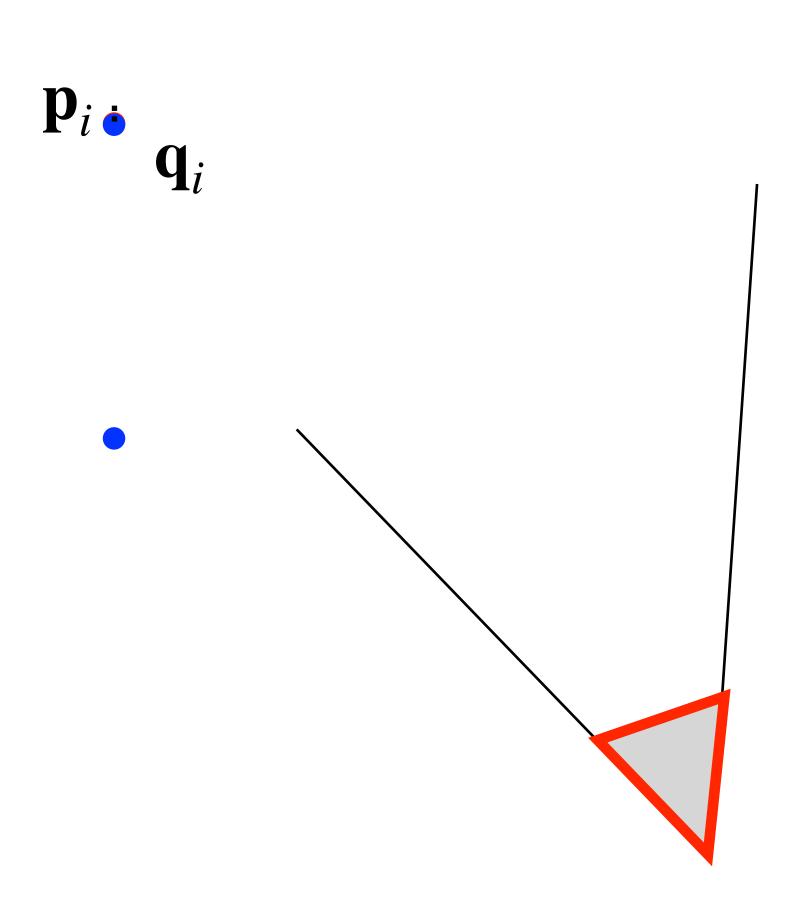
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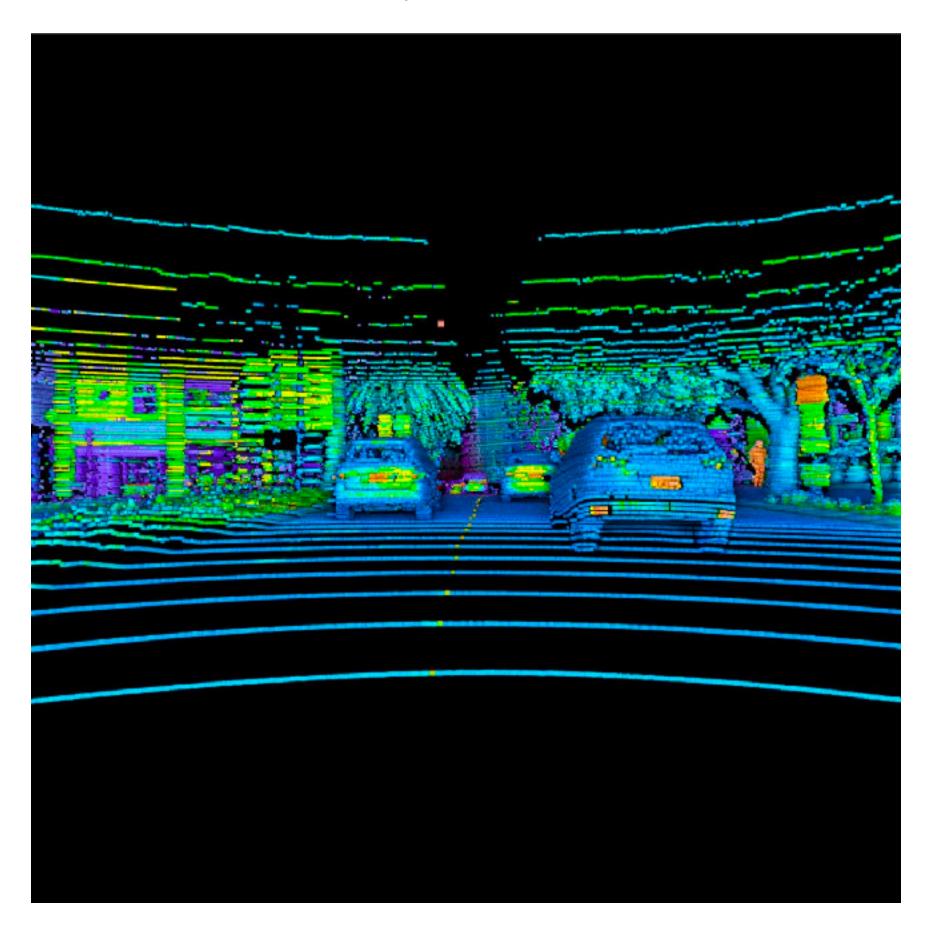


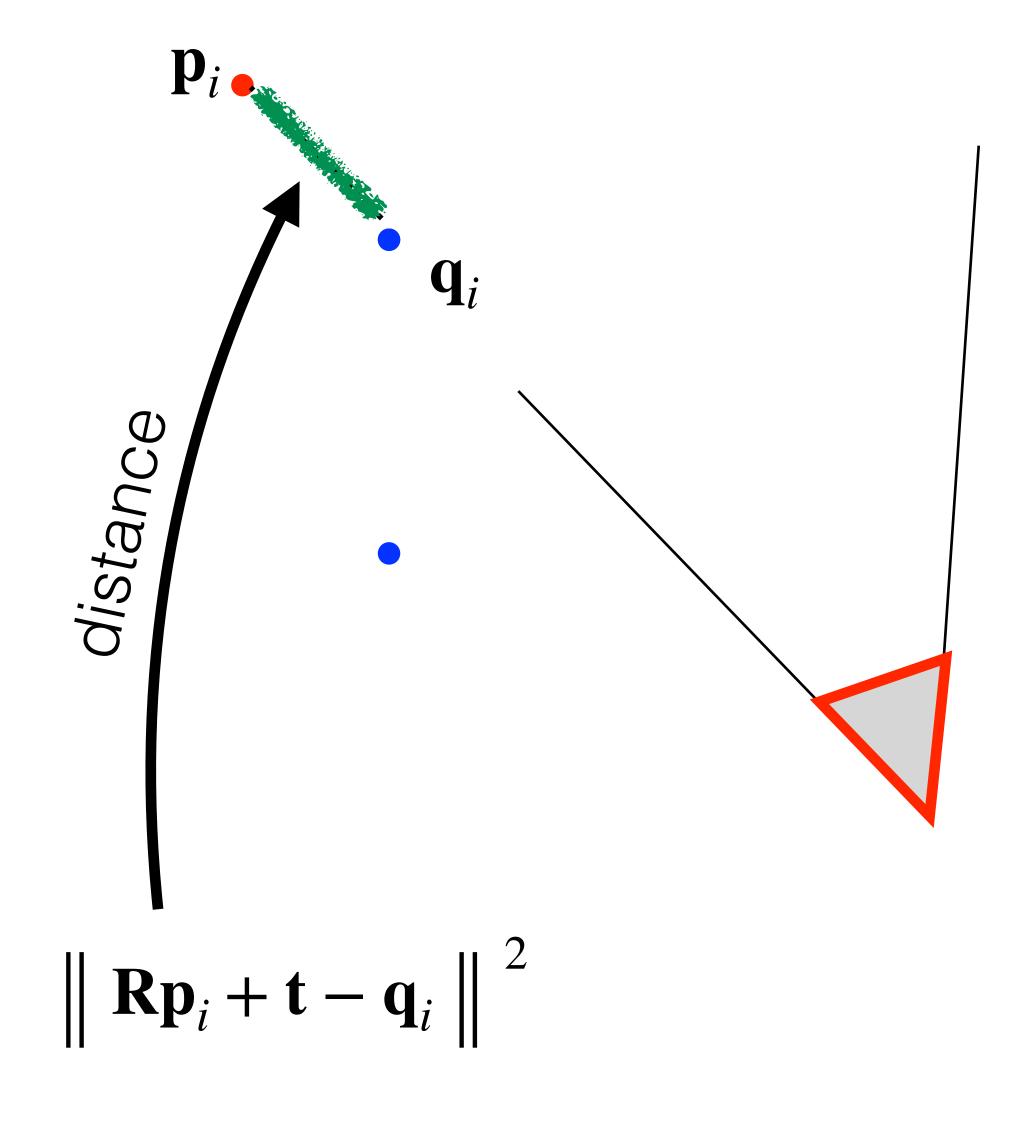
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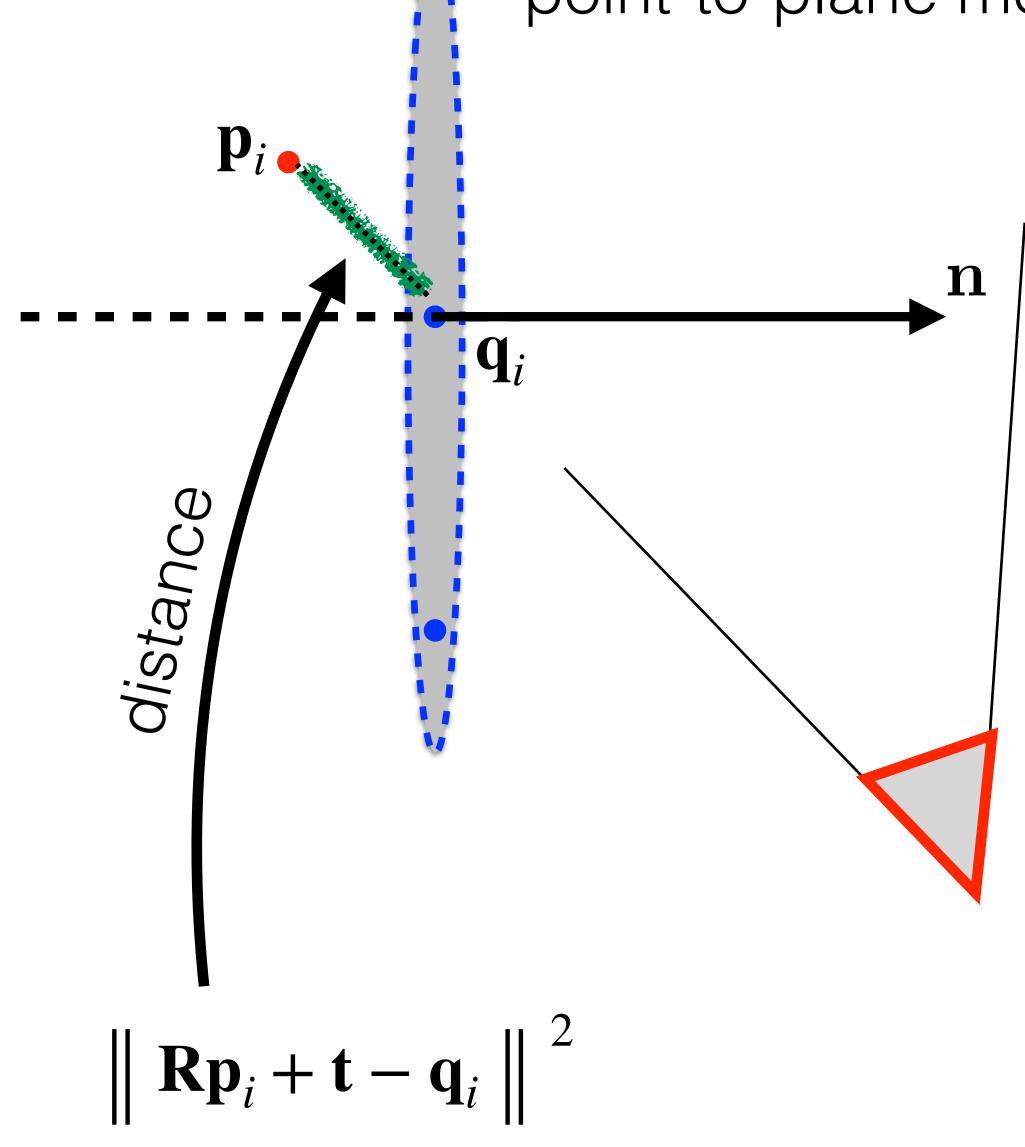


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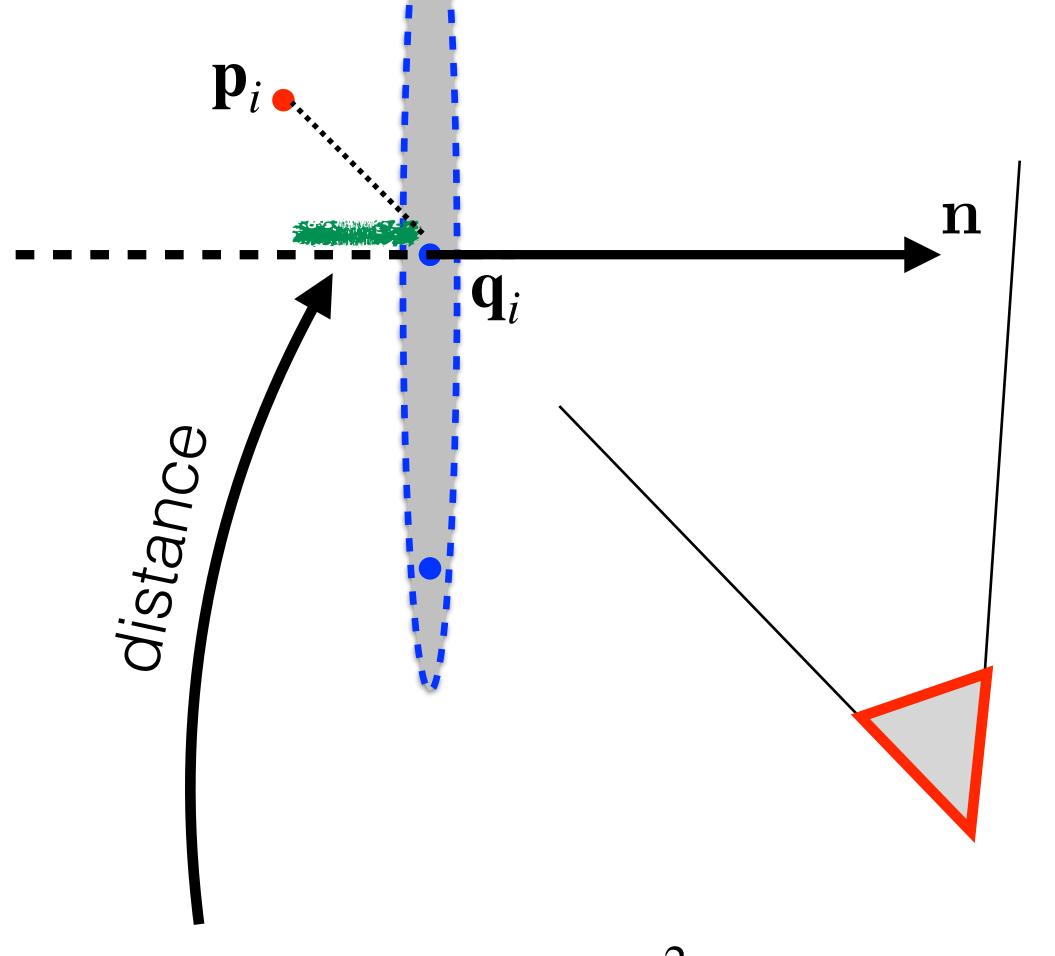




$$\mathbf{R}^{\star}, \mathbf{t}^{\star}, = \underset{\mathbf{R} \in SO(3), \mathbf{t}}{\operatorname{arg min}} \sum_{i} \left\| \mathbf{R} \mathbf{p}_{i} + \mathbf{t} - \mathbf{q}_{i} \right\|^{2}$$



$$\mathbf{R}^{\star}, \mathbf{t}^{\star}, = \underset{\mathbf{R} \in SO(3), \mathbf{t}}{\operatorname{arg min}} \sum_{i} \| \mathbf{R} \mathbf{p}_{i} + \mathbf{t} - \mathbf{q}_{i} \|^{2}$$

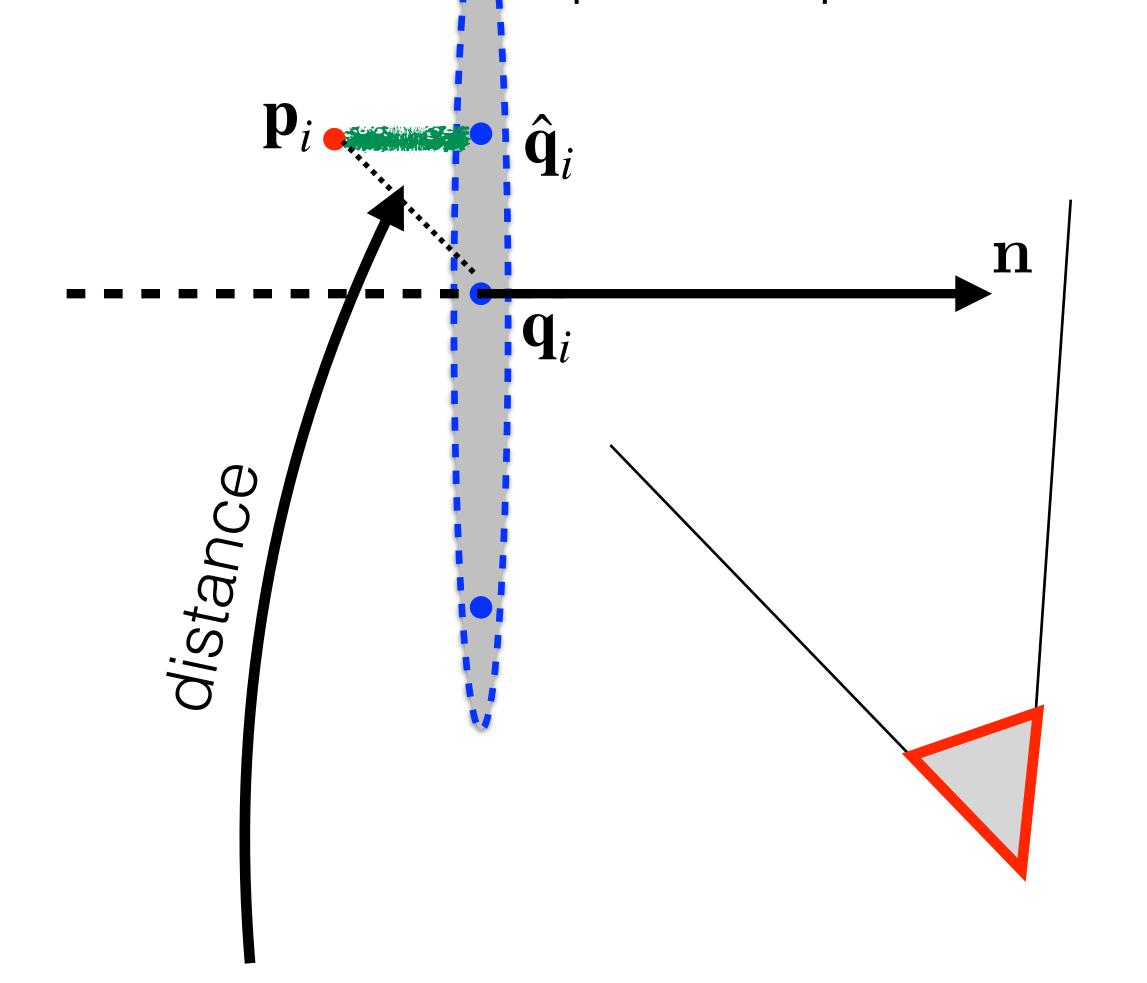


Absolute orientation:

$$\mathbf{P}_{i}^{\star}$$
, $\mathbf{R} \in SO(3)$, \mathbf{E}_{i}^{\star}

$$\mathbf{R}^{\star}, \mathbf{t}^{\star}, = \underset{\mathbf{R} \in SO(3), \mathbf{t}}{\operatorname{arg min}} \sum_{i} \left\| \mathbf{n}^{\mathsf{T}} (\mathbf{R} \mathbf{p}_{i} + \mathbf{t} - \mathbf{q}_{i}) \right\|^{2}$$

Does not have closed-form solution



Absolute orientation:

$$\mathbf{P}_{i}^{\star}, \mathbf{t}^{\star}, \mathbf{r}_{s}^{\star}$$
 arg min \mathbf{P}_{i} \mathbf{P}_{i} \mathbf{q}_{i} $\mathbf{R} \in SO(3), \mathbf{t}$

$$\mathbf{R}^{\star}, \mathbf{t}^{\star}, = \underset{\mathbf{R} \in SO(3), \mathbf{t}}{\operatorname{arg} \operatorname{min}} \sum_{i} \|\mathbf{n}^{\mathsf{T}}(\mathbf{R}\mathbf{p}_{i} + \mathbf{t} + \mathbf{q}_{i})\|^{2}$$

$$\mathbf{R}^{\star}, \mathbf{t}^{\star}, = \underset{\mathbf{R} \in SO(3), \mathbf{t}}{\operatorname{arg \, min}} \sum_{i} \| \mathbf{R} \mathbf{p}_{i} + \mathbf{t} - \hat{\mathbf{q}}_{i} \|^{2}$$

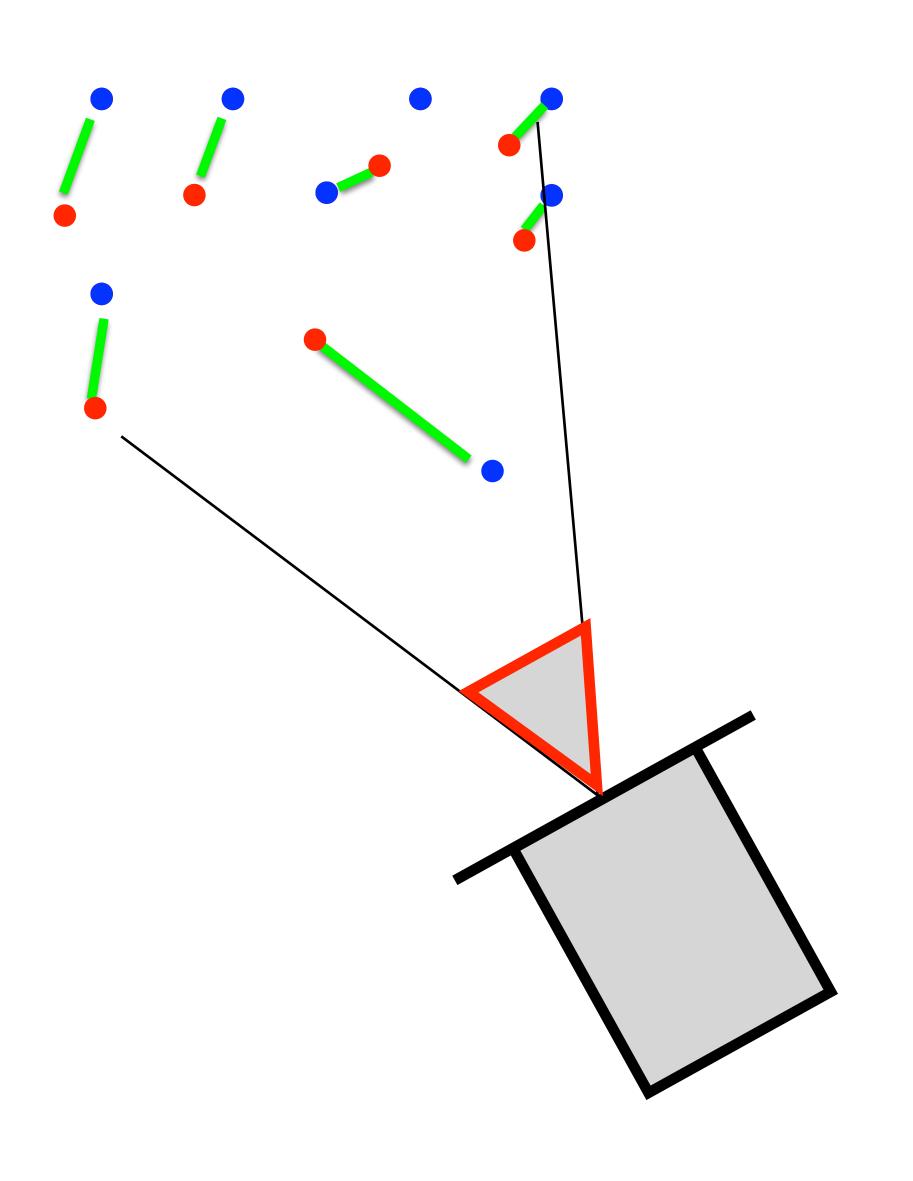
$$\|\mathbf{R}\mathbf{p}_i + \mathbf{t} - \hat{\mathbf{q}}_i\|^2$$

Avoid gradient optimization => create virtual map point $\hat{\mathbf{q}}_i$

- Converges to a local minima with unknown region of convergence
 => good initialization needed
- Fails in degenerate environments
 (e.g. forward motion in long hallways, or rotation in circular room)
 => other models (point2plane) and sensors (camera) often introduced.

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 (e.g. forward motion in long hallways, or rotation in circular room)
 => other models (point2plane) and sensors (camera) often introduced.

Alignment quality is determined by the quality of correspondences



Compatibility measure:

- Lidar
 - normals
 - curvature
 - any measure of shape similarity
- Camera
 - colors
 - semantic consistency (car-car)
 - any measure of visual similarity
 - dynamic object suppression

- Converges to a local minima with unknown region of convergence
 => good initialization needed
- Fails in degenerate environments
 (e.g. forward motion in long hallways, or rotation in circular room)
 => other models (point2plane) and sensors (camera) often introduced.
- Pointcloud map is not suitable for planning
 - => better representation (e.g. occupancy grid)

Maps

- 2D/3D pointcloud map
- 2D/3D Occupancy grid
- 2.5D map (hightmap)
- Surfel/feature map
- Semantic map
- Cost map
- Traversability map
- Topological map
- Functional map



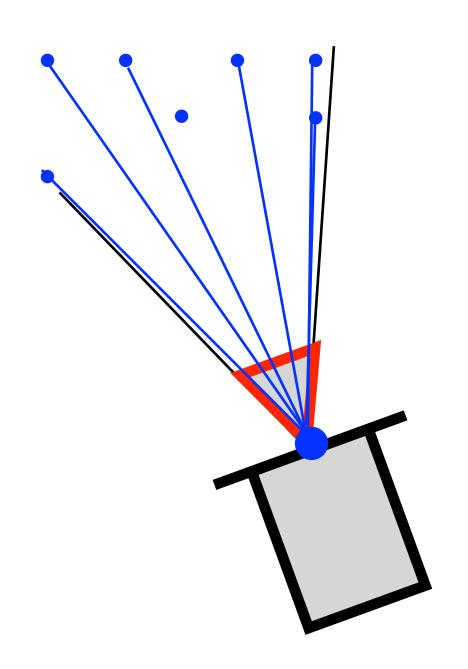


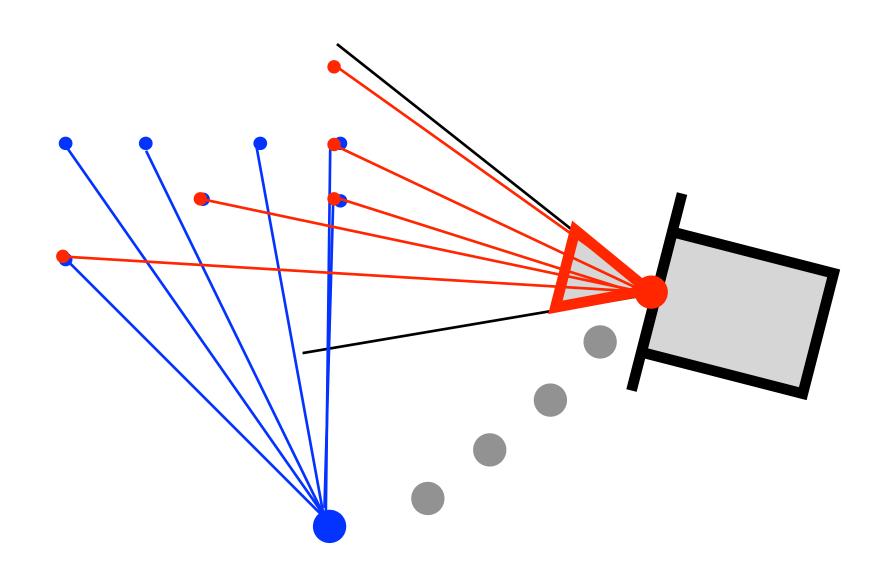
Abstract (Everyone has its own understanding)

Naive: If pointcloud map is available you can use ICP to align wrt existing map => abs. loc

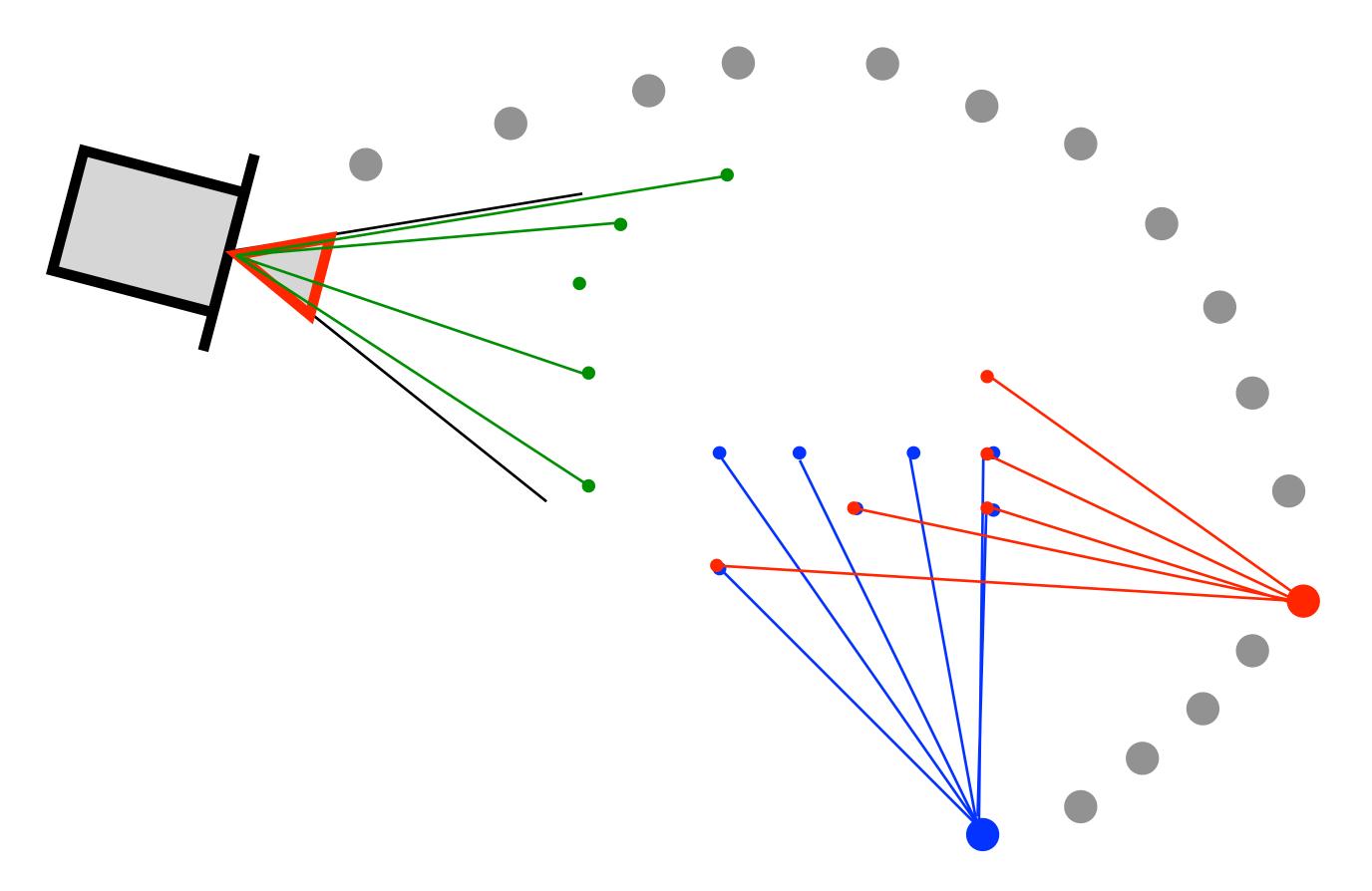
Drawback and advantages?

Less naive: combine the map with the factorgraph

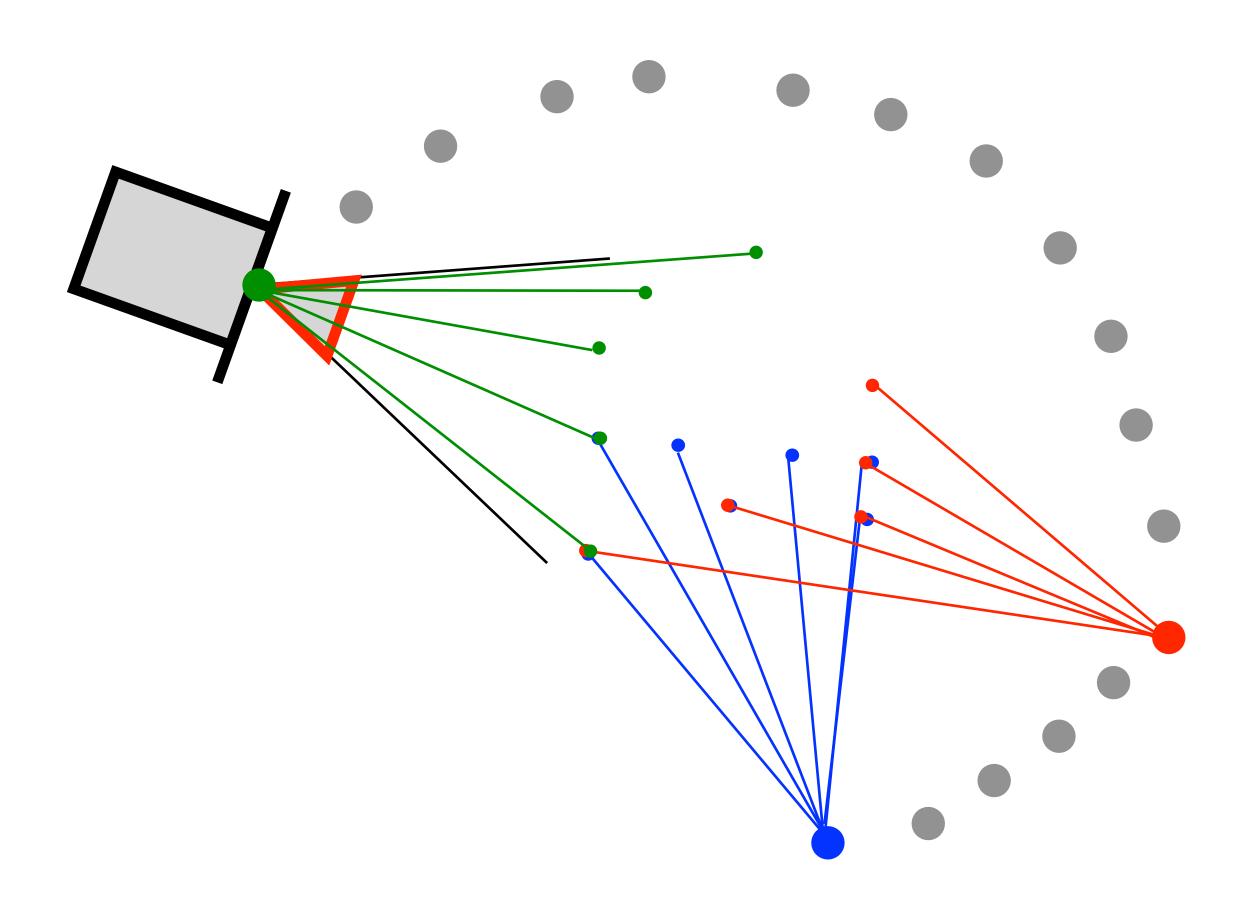




Some poses connected with its own pointcloud (keyframes)
When factorgraph optimized the current map is union of keyframes



Some poses connected with its own pointcloud (keyframes)
When factorgraph optimized the current map is union of keyframes



Some poses connected with its own pointcloud (keyframes)
When factorgraph optimized the current map is union of keyframes

Occupancy grid

- Exploration requires to distinguish
 - "occupied"/"unoccupied" assess traversability
 - "unknown" space to motivate the exploration.
- Evently spaced bins with random variables representing its occupancy

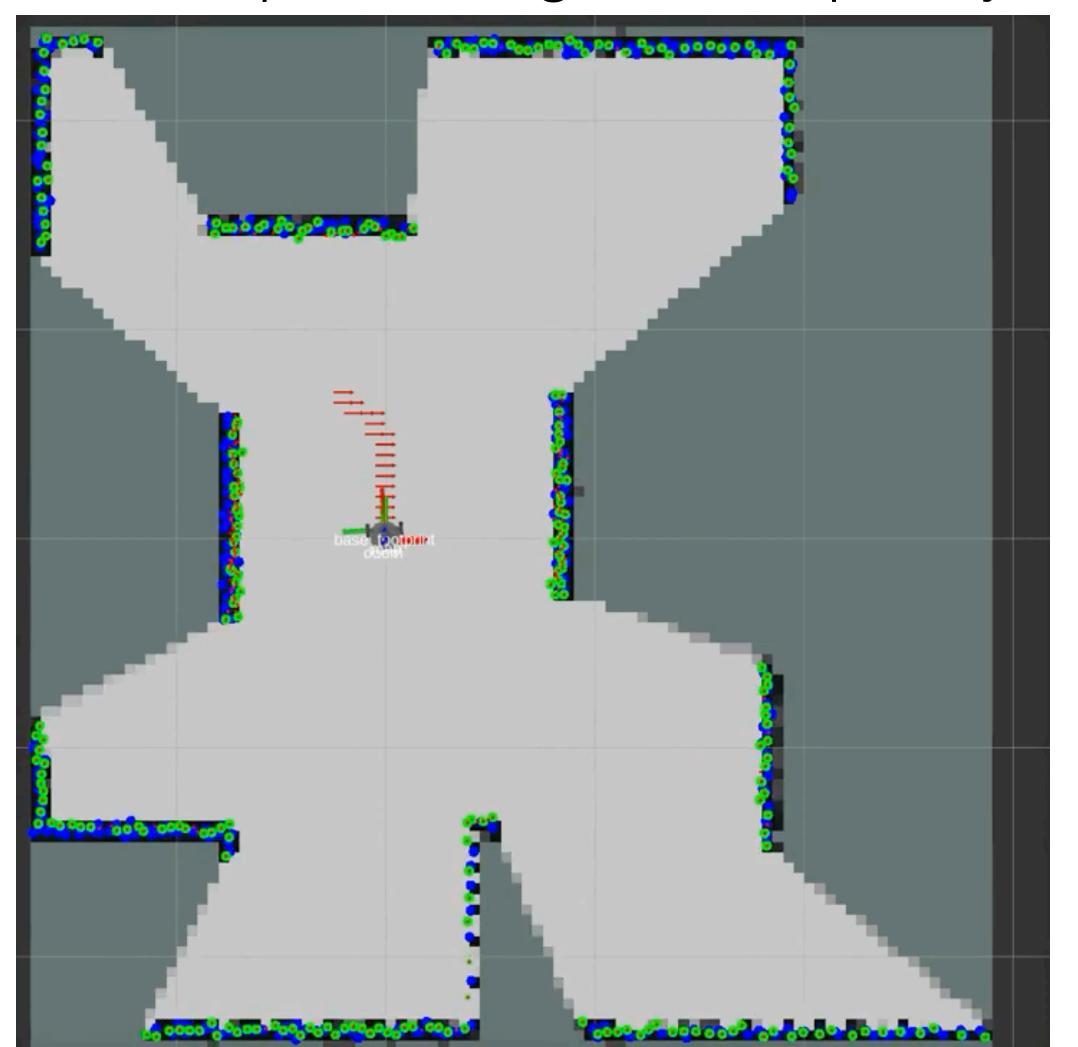
Occupancy grid

- Exploration requires to distinguish
 - "occupied"/"unoccupied" assess traversability
 - "unknown" space to motivate the exploration.
- Evently spaced bins with random variables representing its occupancy
- occupied (+1)
- unknown (0)
- unoccupied (-1)

Memory requirements:

 $10x10cm bins => 10^8 bins/km^2$ $1byte/bin => 0.1GB/km^2$

Octomap representation



- Converges to a local minima with unknown region of convergence
 => good initialization needed
- Measurement probability model fails in degenerate environments
 (e.g. forward motion in long hallways, or rotation in circular room)
 => other models (point2plane) and sensors (camera) often introduced.
- Pointcloud map is not suitable for planning
 better representation (e.g. occupancy grid)
- Next: Normal distribution sensitive to outliers due to L2 norm minimization
 => RANSAC