AI CENTER
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Quantum Computing

Jakub Mareček, with
Johannes Aspman, Georgios Korpas, Germán Matilla, and Waqas Parvaiz

Faculty of Electrical Engineering
Czech Technical University in Prague

February 23rd, 2024

Quantum Computing

1. Motivation: “A social phenomenon”
2. Motivation: Opportunities and Limitations
3. Organization of the Course
4. Qubits and How to Implement them
5. A Theoretical Computer Science Point of View
6. Three Use Cases in Financial Services

Quantum Computing: A Short History

- 1965: Nobel prize for Richard P. Feynman.
- 1973: Alexander Holevo publishes a paper showing that n qubits can carry more than n classical bits of information, but at most n classical bits are accessible.
- 1973: Charles H. Bennett publishes papers on reversible computing.
- 1980: Tommaso Toffoli introduces the Toffoli gate, which is a key element in both classical reversible computing and quantum computing.
- 1980: Paul Benioff and Yuri Manin publish papers on quantum computing.
- 1981: At the “First Conference on the Physics of Computation,” Paul Benioff and Richard Feynman give talks on quantum computing.
- 1985: David Deutsch introduces the first universal model of quantum computing.
- 1993: Dan Simon suggests the so-called Simon's problem, for which a quantum computer could be exponentially faster than a conventional computer (under mild assumptions on the oracles).
- 1994: Peter Shor extends Simon’s work to Shor's algorithm for factoring integers.
- 1998: A team incl. Isaac L. Chuang demonstrates a 2-qubit NMR-based quantum computer.
- 2022: Nobel prize for Alain Aspect, John F. Clauser and Anton Zeilinger.

Quantum Computing: A Social Phenomenon

- Feynman (1986): "Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy." ("Proof by authority")
- A prototypical problem: Computing the ground-state energy (eigenvalue of the fermionic Hamiltonian), usually discretized into a basis (of size L). One needs to restrict oneself to "generic" molecules and materials.
- Seth Lloyd (1996): Exponential quantum advantage conjecture
- Kitaev (2003): Ground state characterization is QMA (cf. the Ising Hamiltonian)

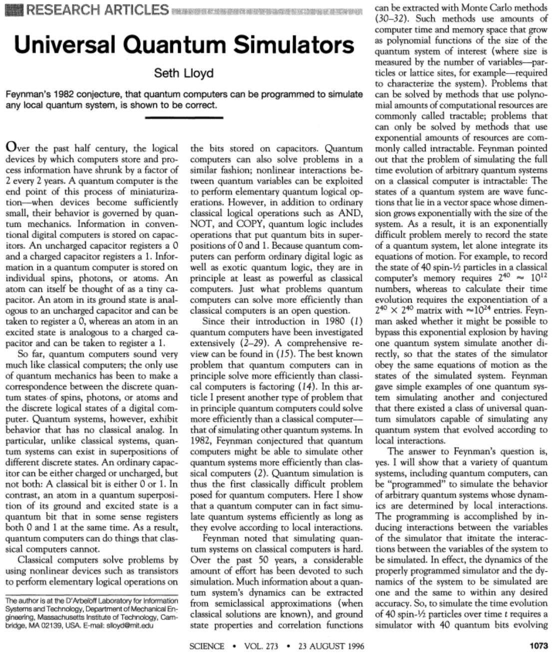
https://www.ams.org/books/gsm/047/

https://www.science.org/doi/abs/10.1126/science.273.5278.1073

https://arxiv.org/abs/quant-ph/0302079 https://arxiv.org/abs/quant-ph/0406180v2

https://journals.aps.org/prxquantum/abstract/10.1103/PRXQuantum.3.010318

https://simons.berkeley.edu/events/quantum-colloquium-there-evidence-exponential-quantum-advantage-quantum-chemistry



RESEARCH ARTICLES

Universal Quantum Simulators

Seth Lloyd

Feynman's 1982 conjecture, that quantum computers can be programmed to simulate any local quantum system, is shown to be correct.

Over the past half century, the logical devices by which computers store and process information have shrunk by a factor of 2 every 2 years. A quantum computer is the end point of this process of miniaturization—when devices become sufficiently small, their behavior is governed by quantum mechanics. Information in conventional digital computers is stored on capacitors. An uncharged capacitor registers a 0 and a charged capacitor registers a 1. Information in a quantum computer is stored on individual spins, photons, or atoms. An atom can itself be thought of as a tiny capacitor. An atom in its ground state is analogous to an uncharged capacitor and can be taken to register a 0, whereas an atom in an excited state is analogous to a charged capacitor and can be taken to register a 1.

So far, quantum computers sound very much like classical computers; the only use of quantum mechanics has been to make a correspondence between the discrete quantum states of spins, photons, or atoms and the discrete logical states of a digital computer. Quantum systems, however, exhibit behavior that has no classical analog. In particular, unlike classical systems, quantum systems can exist in superpositions of different discrete states. An ordinary capacitor can be either charged or uncharged, but not both: A classical bit is either 0 or 1. In contrast, an atom in a quantum superposition of its ground and excited state is a quantum bit that in some sense registers both 0 and 1 at the same time. As a result, quantum computers can do things that classical computers cannot.

Classical computers solve problems by using nonlinear devices such as transistors to perform elementary logical operations on the bits stored on capacitors. Quantum computers can also solve problems in a similar fashion; nonlinear interactions between quantum variables can be exploited to perform elementary quantum logical operations. However, in addition to ordinary classical logical operations such as AND, NOT, and COPY, quantum logic includes operations that put quantum bits in superpositions of 0 and 1. Because quantum computers can perform ordinary digital logic as well as exotic quantum logic, they are in principle at least as powerful as classical computers. Just what problems quantum computers can solve more efficiently than classical computers is an open question.

Since their introduction in 1980 (*1*) quantum computers have been investigated extensively (*2–29*). A comprehensive review can be found in (*15*). The best known problem that quantum computers can in principle solve more efficiently than classical computers is factoring (*14*). In this article I present another type of problem that in principle quantum computers could solve more efficiently than a classical computer—that of simulating other quantum systems. In 1982, Feynman conjectured that quantum computers might be able to simulate other quantum systems more efficiently than classical computers (*2*). Quantum simulation is thus the first classically difficult problem posed for quantum computers. Here I show that a quantum computer can in fact simulate quantum systems efficiently as long as they evolve according to local interactions.

Feynman noted that simulating quantum systems on classical computers is hard. Over the past 50 years, a considerable amount of effort has been devoted to such simulation. Much information about a quantum system's dynamics can be extracted from semiclassical approximations (when classical solutions are known), and ground state properties and correlation functions can be extracted with Monte Carlo methods (*30–32*). Such methods use amounts of computer time and memory space that grow as polynomial functions of the size of the quantum system of interest (where size is measured by the number of variables—particles or lattice sites, for example—required to characterize the system). Problems that can be solved by methods that use polynomial amounts of computational resources are commonly called tractable; problems that can only be solved by methods that use exponential amounts of resources are commonly called intractable. Feynman pointed out that the problem of simulating the full time evolution of arbitrary quantum systems on a classical computer is intractable: The states of a quantum system are wave functions that lie in a vector space whose dimension grows exponentially with the size of the system. As a result, it is an exponentially difficult problem merely to record the state of a quantum system, let alone integrate its equations of motion. For example, to record the state of 40 spin-½ particles in a classical computer's memory requires $2^{40} \approx 10^{12}$ numbers, whereas to calculate their time evolution requires the exponentiation of a $2^{40} \times 2^{40}$ matrix with $\approx 10^{24}$ entries. Feynman asked whether it might be possible to bypass this exponential explosion by having one quantum system simulate another directly, so that the states of the simulator obey the same equations of motion as the states of the simulated system. Feynman gave simple examples of one quantum system simulating another and conjectured that there existed a class of universal quantum simulators capable of simulating any quantum system that evolved according to local interactions.

The answer to Feynman's question is, yes. I will show that a variety of quantum systems, including quantum computers, can be "programmed" to simulate the behavior of arbitrary quantum systems whose dynamics are determined by local interactions. The programming is accomplished by inducing interactions between the variables of the simulator that imitate the interactions between the variables of the system to be simulated. In effect, the dynamics of the properly programmed simulator and the dynamics of the system to be simulated are one and the same to within any desired accuracy. So, to simulate the time evolution of 40 spin-½ particles over time t requires a simulator with 40 quantum bits evolving

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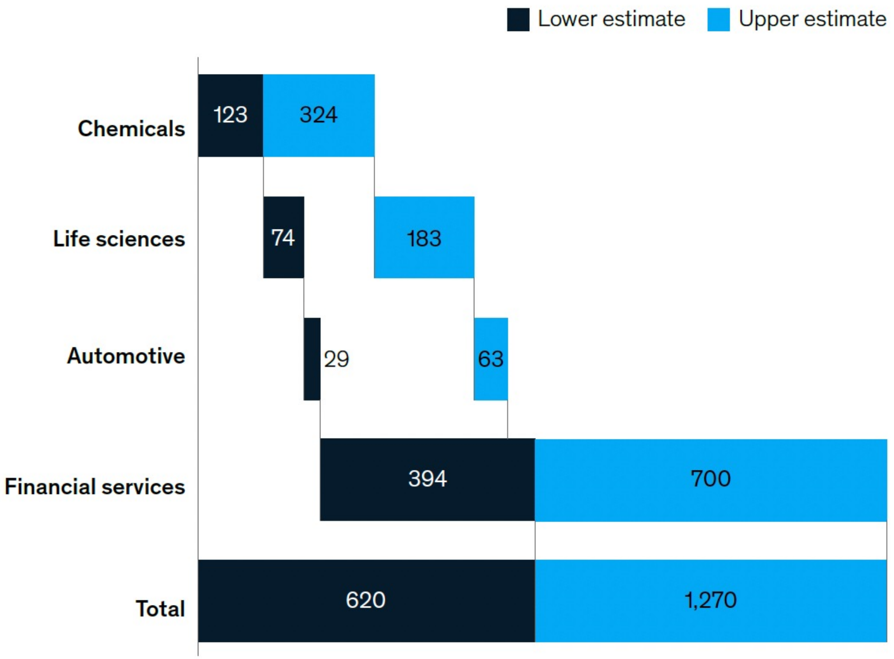
Quantum Computing: A Social Phenomenon

- McKinsey estimates and recommendations to CEOs:

Four industries expected to see first impact

Value at stake with incremental impact of QC by 2035, $ billion



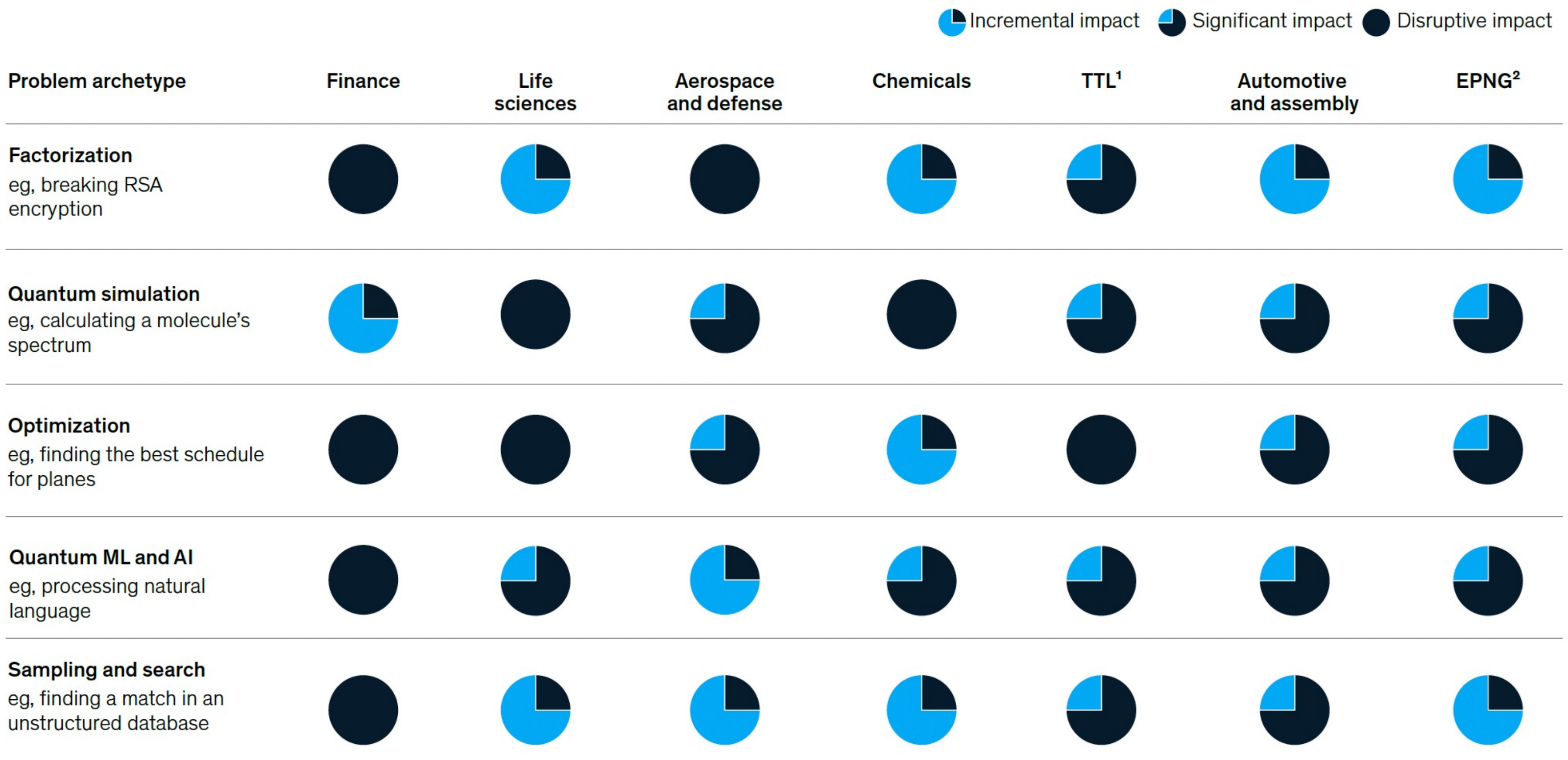
1. Follow industry developments and actively screen quantum-computing use cases with an in-house team of quantum-computing experts or by collaborating with industry entities and by joining a quantum-computing consortium.
2. Understand the most significant risks and disruptions and opportunities in their industries.
3. Consider whether to partner with or invest in quantum-computing players—mostly software—to facilitate access to knowledge and talent.
4. Consider recruiting in-house quantum-computing talent. Even a small team of up to three experts may be enough to help an organization explore possible use cases and screen potential strategic investments in quantum computing.
5. Prepare by building digital infrastructure that can meet the basic operating demands of quantum computing; make relevant data available in digital databases and set up conventional computing workflows to be quantum ready once more powerful quantum hardware becomes available.

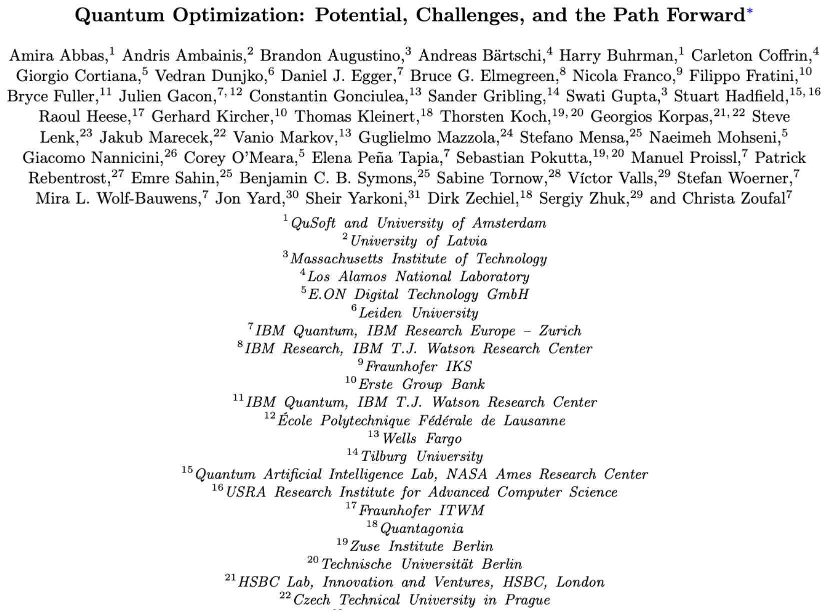
https://www.mckinsey.com/capabilities/mckinsey-digital/our-insights/quantum-technology-sees-record-investments-progress-on-talent-gap#/

https://www.mckinsey.com/business-functions/mckinsey-digital/our-insights/quantum-computing-use-cases-are-getting-real-what-you-need-to-know

Quantum Computing: A Social Phenomenon

- McKinsey estimates and recommendations to CEOs vs. our expert opinion:





Quantum Optimization: Potential, Challenges, and the Path Forward*

Amira Abbas,[1] Andris Ambainis,[2] Brandon Augustino,[3] Andreas Bärtschi,[4] Harry Buhrman,[1] Carleton Coffrin,[4] Giorgio Cortiana,[5] Vedran Dunjko,[6] Daniel J. Egger,[7] Bruce G. Elmegreen,[8] Nicola Franco,[9] Filippo Fratini,[10] Bryce Fuller,[11] Julien Gacon,[7,12] Constantin Gonciulea,[13] Sander Gribling,[14] Swati Gupta,[3] Stuart Hadfield,[15,16] Raoul Heese,[17] Gerhard Kircher,[10] Thomas Kleinert,[18] Thorsten Koch,[19,20] Georgios Korpas,[21,22] Steve Lenk,[23] Jakub Marecek,[22] Vanio Markov,[13] Guglielmo Mazzola,[24] Stefano Mensa,[25] Naeimeh Mohseni,[5] Giacomo Nannicini,[26] Corey O'Meara,[5] Elena Peña Tapia,[7] Sebastian Pokutta,[19,20] Manuel Proissl,[7] Patrick Rebentrost,[27] Emre Sahin,[25] Benjamin C. B. Symons,[25] Sabine Tornow,[28] Víctor Valls,[29] Stefan Woerner,[7] Mira L. Wolf-Bauwens,[7] Jon Yard,[30] Sheir Yarkoni,[31] Dirk Zechiel,[18] Sergiy Zhuk,[29] and Christa Zoufal[7]

[1] *QuSoft and University of Amsterdam*
[2] *University of Latvia*
[3] *Massachusetts Institute of Technology*
[4] *Los Alamos National Laboratory*
[5] *E.ON Digital Technology GmbH*
[6] *Leiden University*
[7] *IBM Quantum, IBM Research Europe – Zurich*
[8] *IBM Research, IBM T.J. Watson Research Center*
[9] *Fraunhofer IKS*
[10] *Erste Group Bank*
[11] *IBM Quantum, IBM T.J. Watson Research Center*
[12] *École Polytechnique Fédérale de Lausanne*
[13] *Wells Fargo*
[14] *Tilburg University*
[15] *Quantum Artificial Intelligence Lab, NASA Ames Research Center*
[16] *USRA Research Institute for Advanced Computer Science*
[17] *Fraunhofer ITWM*
[18] *Quantagonia*
[19] *Zuse Institute Berlin*
[20] *Technische Universität Berlin*
[21] *HSBC Lab, Innovation and Ventures, HSBC, London*
[22] *Czech Technical University in Prague*

https://www.mckinsey.com/capabilities/mckinsey-digital/our-insights/quantum-technology-sees-record-investments-progress-on-talent-gap#/

https://arxiv.org/pdf/2312.02279.pdf

Quantum Computing: A Social Phenomenon

American Banker:

- 25% of financial institutions already invest in quantum
- 45% plan to invest in 2023

Gartner:

- 40% of large companies are planning to create initiatives around quantum computing by 2025.

AMERICAN BANKER

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BANKING POLICY PAYMENTS TECH CREDIT UNIONS WORKPLACE OPINION

TECHNOLOGY

How JPMorgan Chase and other banks plan to use quantum computing

By Penny Crosman September 22, 2022, 2:57 p.m. EDT 5 Min Read

Though quantum computing technology is still new, JPMorgan Chase, Ally Bank, Credit Agricole and other banks are actively testing and in some cases using it, according to speakers at the HPC + AI on Wall Street conference in New York this week.

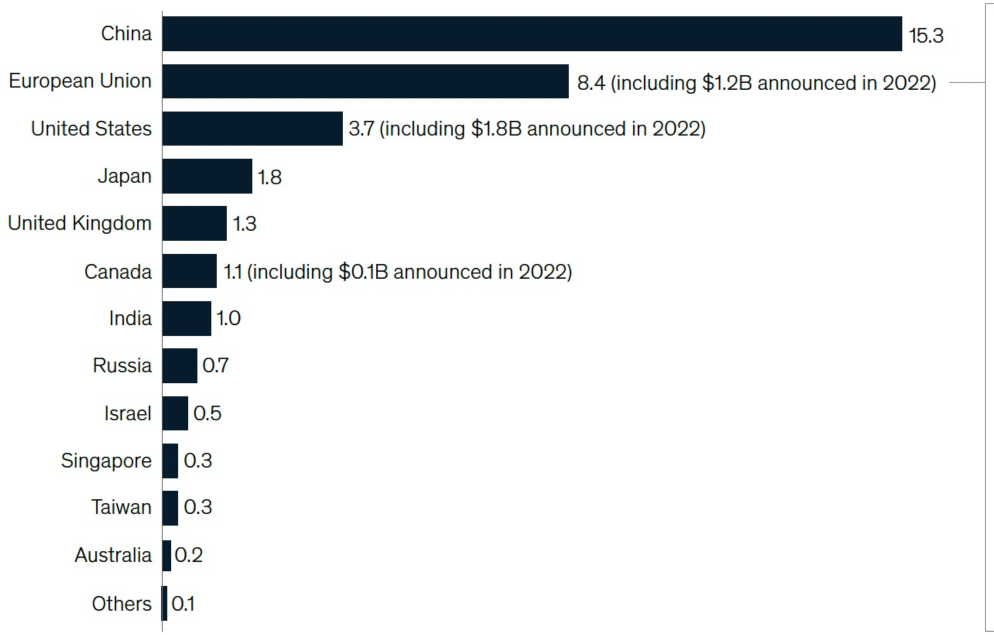
"We realize that if a company doesn't do anything about the market right now, and just waits for quantum advantage to become a reality, when quantum advantage becomes real, it might be too late," said Marco Pistoia, managing director, distinguished engineer, head of global technology applied research and head of quantum computing at JPMorgan Chase. "We want to be ready when quantum advantage becomes possible on a higher level."

https://www.americanbanker.com/news/how-jp-morgan-chase-and-other-banks-plan-to-use-quantum-computing

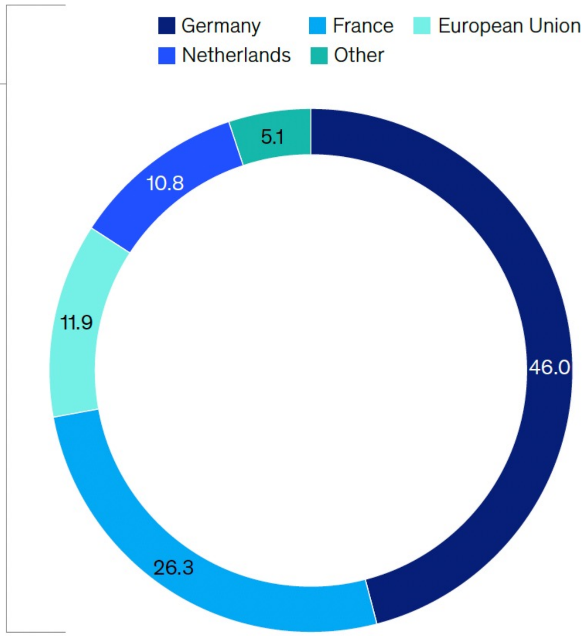
Quantum Computing: A Social Phenomenon

- Circa $80B eco-system
- $30+B of public funding announced

Announced governmental investment,[1] $ billion



EU public investment sources, %



[1] Total historic announced investment; timelines for investment of investment vary per country.

Source: Johnny Kung and Muriam Fancy, *A quantum revolution: Report on global policies for quantum technology*, CIFAR, April 2021; press search

https://www.mckinsey.com/business-functions/mckinsey-digital/our-insights/quantum-computing-use-cases-are-getting-real-what-you-need-to-know

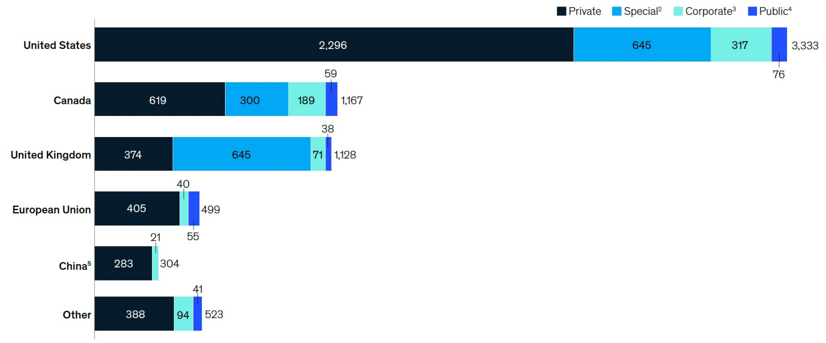
Quantum Computing: A Social Phenomenon

Top 10 venture capital/private equity investments in QT start-ups of all time, by deal size (descending)

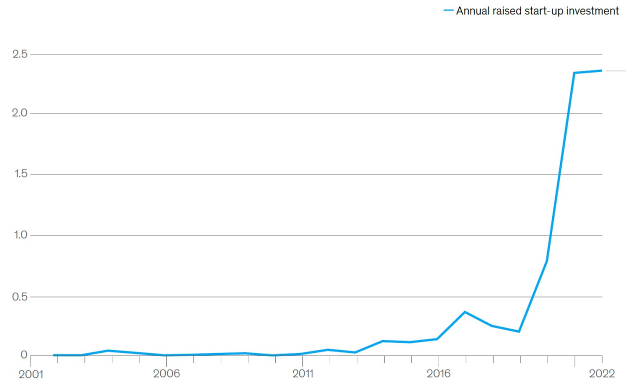
New entrants | Quantum computing | Quantum communications | Quantum sensing

Company	Country	Tech	Segment	Deal size, $ million	Deal year
SandboxAQ	United States	Quantum sensing, Quantum communications, Quantum computing	Application software	500	2022
PsiQuantum	United States	Quantum computing	Hardware manufacturing	450	2021
IonQ	United States	Quantum computing	Hardware manufacturing	350	2021
Rigetti Computing	United States	Quantum computing	Hardware manufacturing	345	2022
Arqit	United Kingdom	Quantum communications	Hardware manufacturing	345	2021
IonQ	United States	Quantum computing	Hardware manufacturing	300	2021
Quantinuum	United Kingdom	Quantum computing	Vertically integrated[1]	300	2021
D-Wave Systems	Canada	Quantum computing	Hardware manufacturing	300	2022
PsiQuantum	United States	Quantum computing	Hardware manufacturing	230	2020
Origin Quantum	China	Quantum computing	Hardware manufacturing	149	2022

Total investment in QT start-ups by location and primary investor type, 2001–22, $ million[1]



Volume of raised investment in the indicated year,[1] $ billion



$2.35B

75%

of investment goes toward quantum computing players

[1]Based on public investment data recorded in PitchBook; actual investment is likely higher.

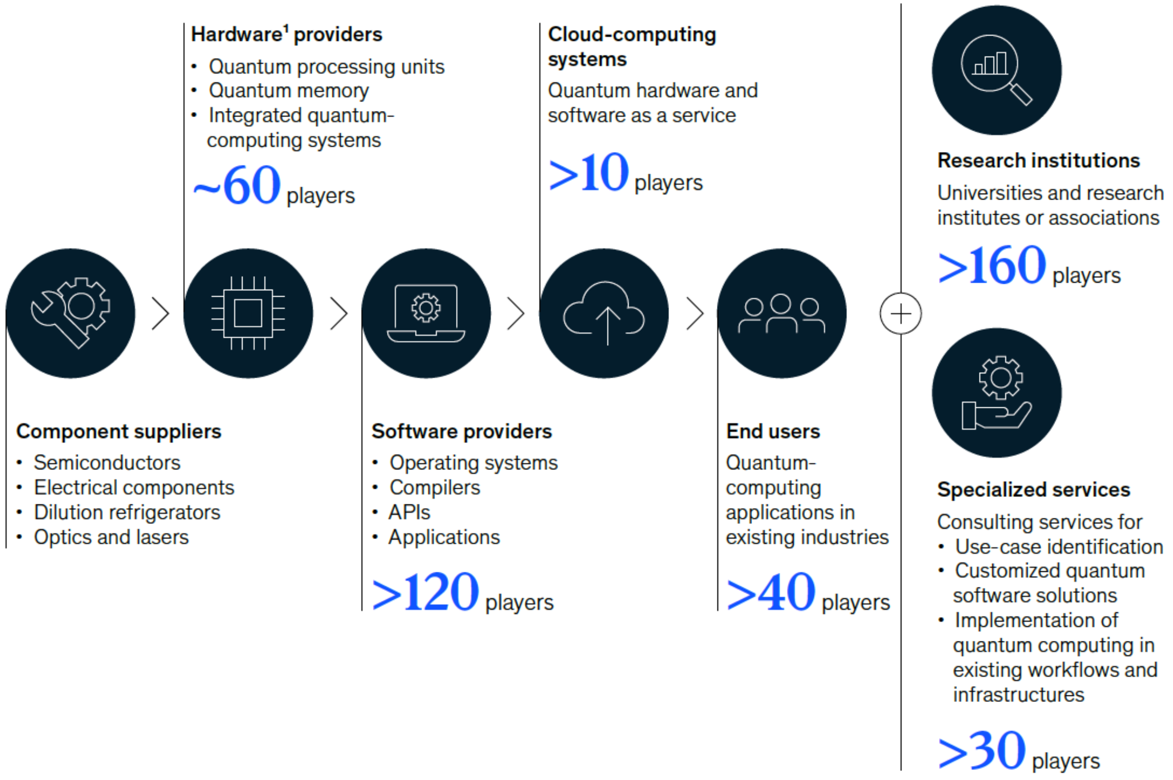
Source: PitchBook

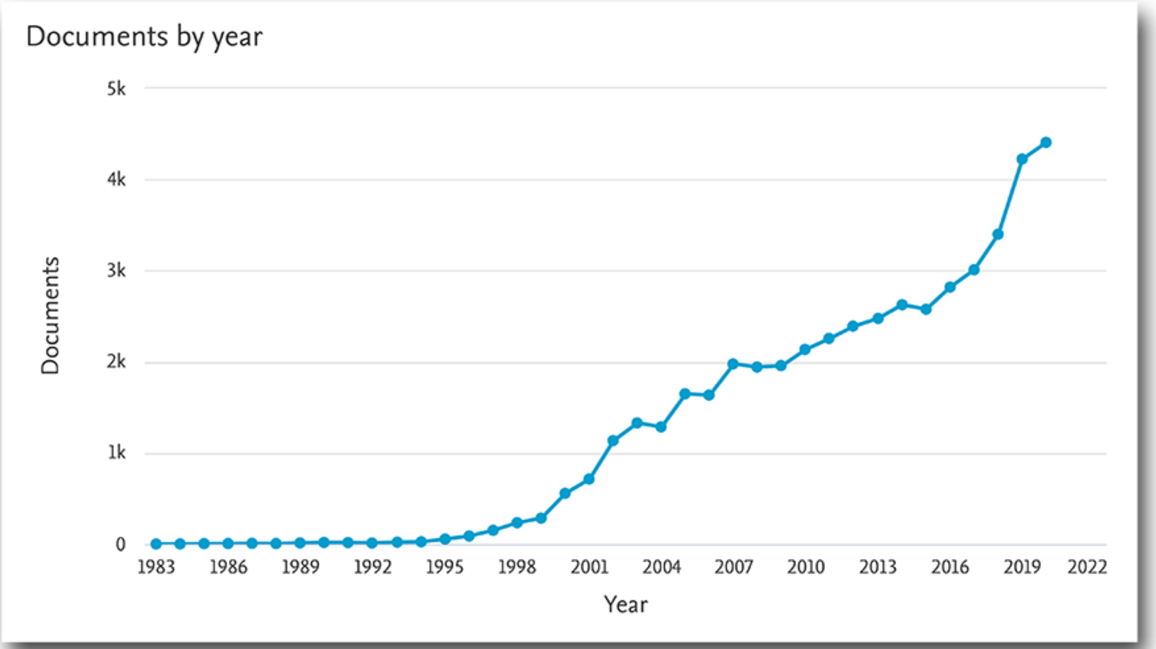
https://www.mckinsey.com/capabilities/mckinsey-digital/our-insights/quantum-technology-sees-record-investments-progress-on-talent-gap#/

Quantum Computing: A Social Phenomenon

In the quantum-computing value chain, software has the largest number of players.

Overview of players in the quantum-computing value chain





https://www.mckinsey.com/business-functions/mckinsey-digital/our-insights/quantum-computing-use-cases-are-getting-real-what-you-need-to-know

https://www.elsevier.com/solutions/scopus/research-and-development/quantum-computing-report

Opportunities

Seen by John Preskill:

- There are problems that **are believed** to be hard for classical computers, but for which quantum algorithms have been discovered that could solve these problems easily under mild assumptions. E.g. factoring.
- Measuring qubits in certain states, which are easy to prepare, samples from a correlated probability distribution that can't be sampled from by any efficient classical means (unless the polynomial hierarchy collapses).
- No known classical algorithm can simulate a quantum computer efficiently.

Seen by yours truly:

- Quantum computers are essentially analog computers, cf. "complexity over the reals", which may violate the "Extended Church-Turing Thesis".

Article | Published: 23 October 2019

Quantum supremacy using a programmable superconducting processor

Frank Arute, Kunal Arya, Ryan Babbush, Dave Bacon, Joseph C. Bardin, Rami Barends, Rupak Biswas, Sergio Boixo, Fernando G. S. L. Brandao, David A. Buell, Brian Burkett, Yu Chen, Zijun Chen, Ben Chiaro, Roberto Collins, William Courtney, Andrew Dunsworth, Edward Farhi, Brooks Foxen, Austin Fowler, Craig Gidney, Marissa Giustina, Rob Graff, Keith Guerin, ... John M. Martinis + Show authors

Nature **574**, 505–510 (2019) | Cite this article

Quantum computational advantage using photons

HAN-SEN ZHONG, HUI WANG, YU-HAO DENG, MING-CHENG CHEN, LI-CHAO PENG, YI-HAN LUO, JIAN QIN, DIAN WU, XING DING, [...], AND JIAN-WEI PAN +14 authors Authors Info & Affiliations

SCIENCE • 3 Dec 2020 • Vol 370, Issue 6523 • pp. 1460-1463 • DOI: 10.1126/science.abe8770

Article | Open Access | Published: 22 February 2023

Suppressing quantum errors by scaling a surface code logical qubit

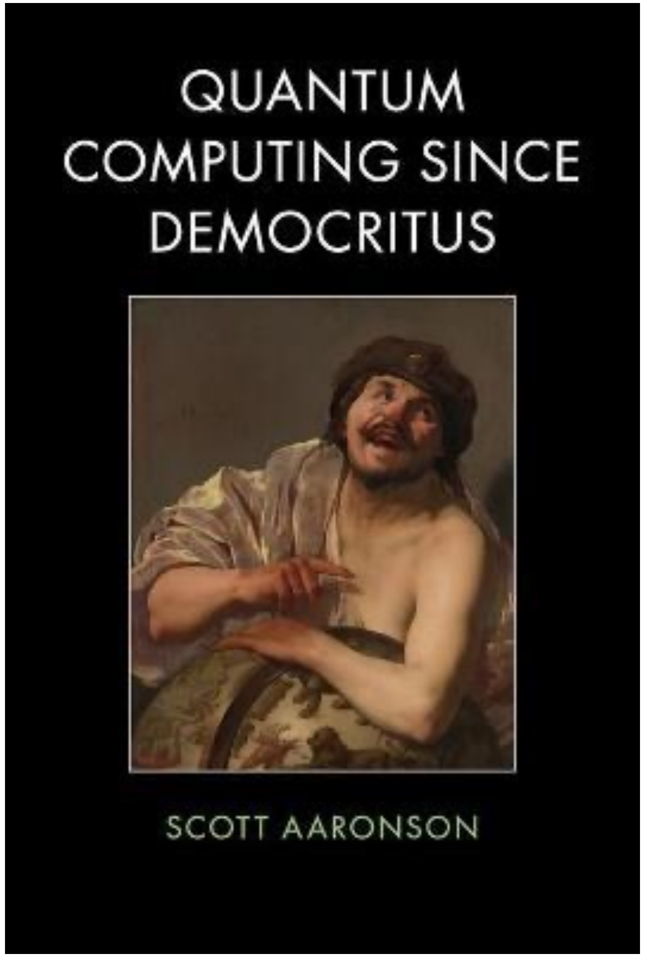
Google Quantum AI

Nature **614**, 676–681 (2023) | Cite this article

https://quantum-journal.org/papers/q-2018-08-06-79/

Eleven Objections of Scott Aaronson

- Works on paper, not in practice.
- Violates Extended Church-Turing Thesis.
- Not enough "real physics."
- Small amplitudes are unphysical.
- Exponentially large states are unphysical.
- Quantum computers are just souped-up analog computers.
- Quantum computers aren't like anything we've ever seen before.
- Quantum mechanics is just an approximation to some deeper theory.
- Decoherence will always be worse than the fault-tolerance threshold.
- We don't need fault-tolerance for classical computers.
- Errors aren't independent.



https://www.scottaaronson.com/democritus/lec14.html

Quantum Computing

1. Motivation: “A social phenomenon”

2. Motivation: Opportunities and Limitations

3. Organization of the Course

4. Qubits and How to Implement them

5. A Theoretical Computer Science point of view

6. Three use cases in financial services

Course Organization: The Team

Who is involved?

Lecturers:

- Bengt Arne **Johannes** Hansson Aspman
- **Jakub** Mareček

Guest speakers:

- **Georgios** Korpas (HSBC)
- **Libor** Caha (TU Munich)

and possibly more (IBM, Sandbox AQ).

Teaching assistants:

- **Germán** Martínez Matilla
- **Waqas** Parvaiz

Course Organization: Syllabus

- 1. Why quantum computing? What is quantum computation good for? The notions of quantum supremacy and quantum advantage. Has Google showcased the former? Why studying quantum computation can also push the limits of classical computation by finding better algorithms or quantum inspired algorithms. The global quantum computing scene. (**Jakub**)
- 2. Broad picture of quantum mechanics. Postulates of quantum mechanics and braket notation. Unitary operators and expectation values. Evolution of quantum states. Classical to quantum bits. The Bloch sphere. Reversible operations on qubits and quantum circuits. State preparation and measurement in quantum mechanics. (**Johannes**)
- 3. Broad overview of computational complexity. Classical Turing machines. The classes P, NP, P-space, Exp. The quantum Turing machine. The classes BQP and QMA. What lies beyond. (**Jakub**)
- 4. Broad overview of classical versus quantum algorithms. Showcase of the exponential speedup of quantum computers using the Deutsch-Josza algorithm. Shor's algorithm, quantum Fourier transform. (**Jakub and Johannes**)
- 5. Grover's algorithm and exponential-time dynamic programming. (**Jakub**)
- 6. Quantum algorithms and quantum random walks. Classical Monte-Carlo and quantum replacements for Monte-Carlo. Applications in Financial Services. (**Georgios**)
- 7. A broad overview of further trends in quantum technologies. Adiabatic computing. Phase estimation. Quantum annealing. Variational algorithms. Quantum Machine Learning. (**Jakub**)

Course Organization: Assessment

- There are 100 points to be collected, which are mapped to grades in the usual fashion (<50b = F, 50-59 = E, ..., 90-100 = A).
- To obtain "zapocet", you need to collect at least 30 points during the term time and attend the exercises. There were more than 60 points on offer last year.
- Up to 40 points are to be collected in a final exam, which can be retaken more than once, if needed.

Homework

Announced 10. 3. 2023, due 24. 3. 2023 (prior to the lecture as a zip file in Brute): qchomework1.pdf

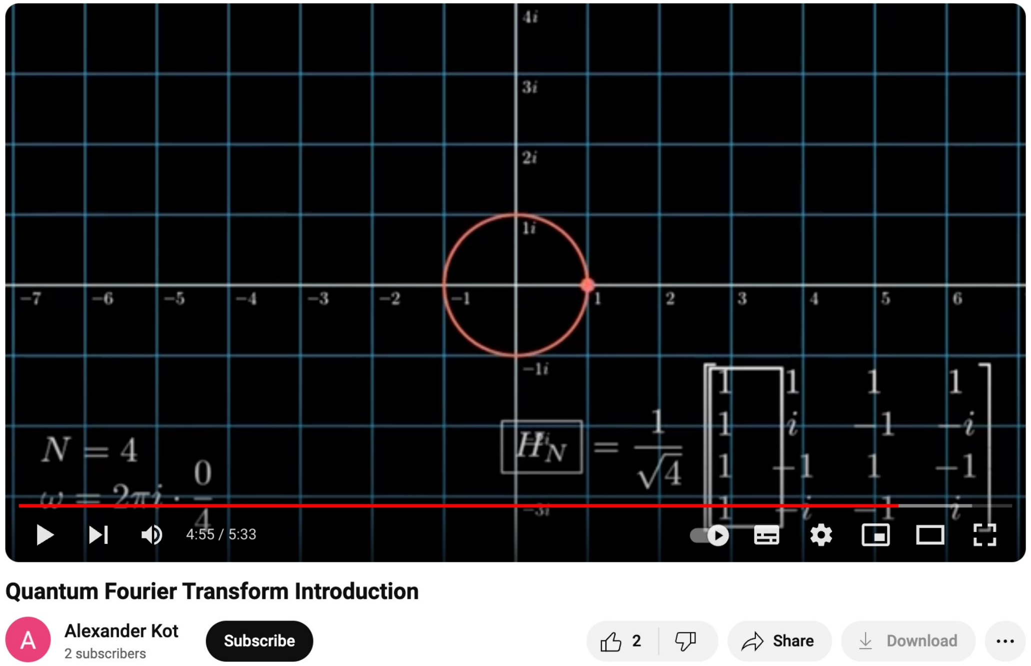
Announced 24. 3. 2023, due 7. 4. 2023 (prior to the lecture as a zip file in Brute): qchomework2.pdf

Announced 2. 4. 2023, due 21. 4. 2023 (prior to the lecture as a zip file in Brute): hw3.pdf

Announced 21. 4. 2023, due 5. 5. 2023 (prior to the lecture as a zip file in Brute): hw4.pdf

Announced 19. 5. 2023, due 2. 6. 2023 (prior to the exam as a zip file in Brute): qchomework5.pdf

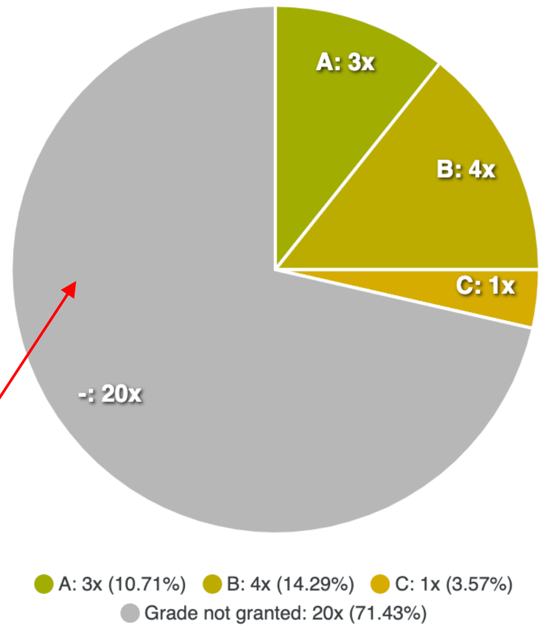
A 20-point project by Alikhan Anuarbekov:



https://www.youtube.com/watch?v=yDDr_wddqYQ

Course Organization: Assessment

- There are 100 points to be collected, which are mapped to grades in the usual fashion (<50b = F, 50-59 = E, ..., 90-100 = A).
- Up to 60 points to be collected during term time (homework and a larger, independent "project").
- To obtain "zapocet", you need to collect at least 30 points during the term time and attend the exercises.
- Up to 40 points are to be collected in a final exam, which can be retaken more than once, if needed.



These students did not show up for the exam.

Course Organization: Resources

Jakub Mareček and Georgios Korpas and Johannes Aspman

Quantum Computing

via Randomized Algorithms

November 10, 2023

Springer

Lectures

24. 2. 2023: Slides

3. 3. 2023: Lecture notes

10. 3. 2023: Lecture notes

17. 3. 2023: Slides, Lecture notes

24. 3. 2023: Slides, Lecture notes

31. 3. 2023: Slides, Lecture notes

6. 4. 2023: Guest lecture of Libor Caha on the quantum advantage with teleportation circuits.

14. 4. 2023: Slides, Lecture notes

21. 4. 2023: Slides, Lecture notes

28. 4. 2023: Slides, Lecture notes

5. 5. 2023: Slides, Lecture notes

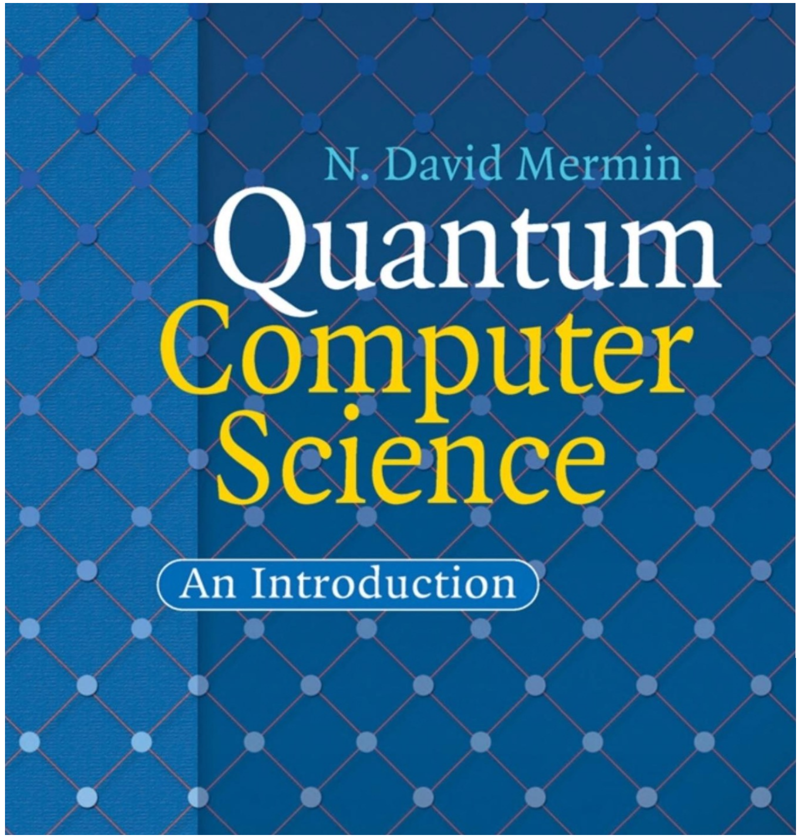
12. 5. 2023: Slides, Lecture notes

19. 5. 2023: Slides, Lecture notes

26. 5. 2023: Guest lecture of Google / SandboxAQ on post-quantum security. (KN:E - 108)

2. 6. 2023: Exam.

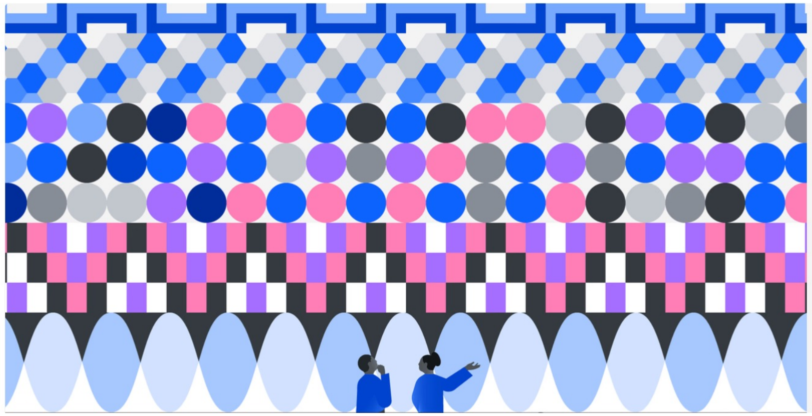
Course Organization: Further Reading



IBM Quantum Learning

Learn the basics of quantum computing, and
how to use IBM Quantum services and systems
to solve real-world problems.

Explore the
latest course



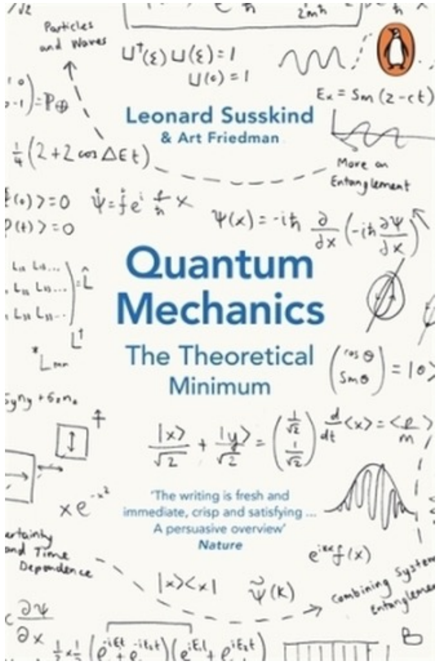
Fundamentals of quantum algorithms

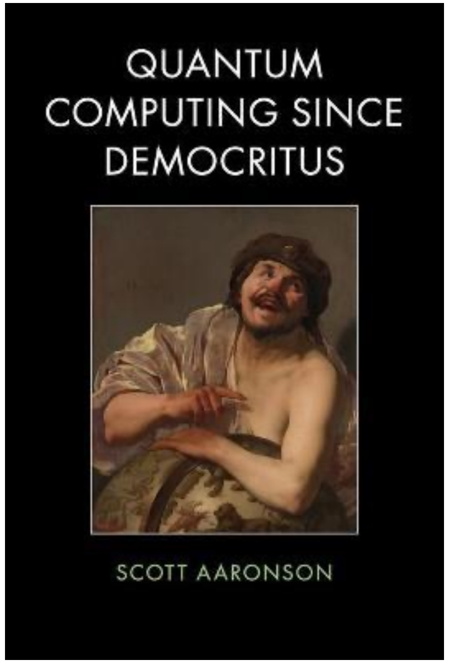
New

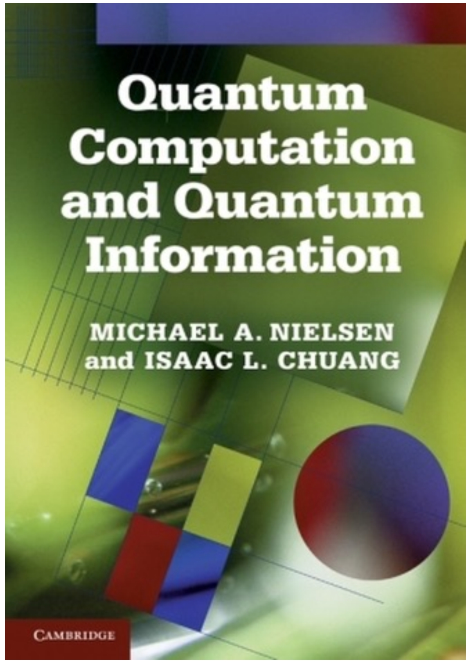
Use quantum computers to solve problems more efficiently, including problems with real-world relevance such as searching and factoring.

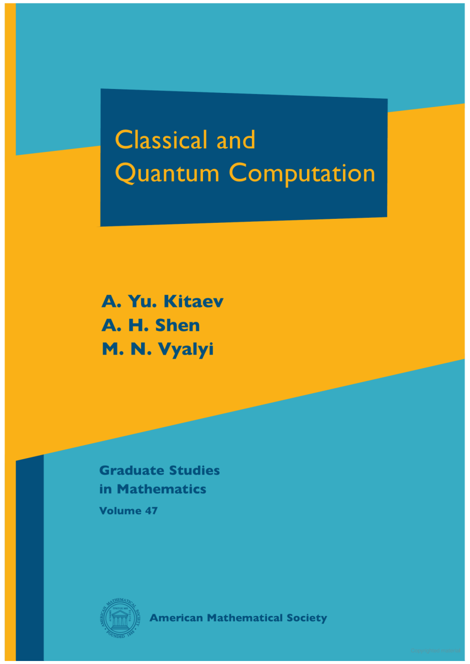
https://learning.quantum.ibm.com/

Course Organization: Further Reading









244 Kc at Megabooks https://www.scottaaronson.com/democritus/ https://www.ams.org/books/gsm/047/

Course Organization: Resources

Quantum Optimization: Potential, Challenges, and the Path Forward*

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[17] *Fraunhofer ITWM*
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[19] *Zuse Institute Berlin*
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(Dated: December 6, 2023)

Recent advances in quantum computers are demonstrating the ability to solve problems at a scale beyond brute force classical simulation. As such, a widespread interest in quantum algorithms has developed in many areas, with optimization being one of the most pronounced domains. Across computer science and physics, there are a number of algorithmic approaches, often with little linkage. This is further complicated by the fragmented nature of the field of mathematical optimization, where major classes of optimization problems, such as combinatorial optimization, convex optimization, non-convex optimization, and stochastic extensions, have devoted communities. With these aspects in mind, this work draws on multiple approaches to study quantum optimization. Provably exact versus heuristic settings are first explained using computational complexity theory — highlighting where quantum advantage is possible in each context. Then, the core building blocks for quantum optimization algorithms are outlined to subsequently define prominent problem classes and identify key open questions that, if answered, will advance the field. The effects of scaling relevant problems on noisy quantum devices are also outlined in detail, alongside meaningful benchmarking problems. We underscore the importance of benchmarking by proposing clear metrics to conduct appropriate comparisons with classical optimization techniques. Lastly, we highlight two domains – finance and sustainability – as rich sources of optimization problems that could be used to benchmark, and eventually validate, the potential real-world impact of quantum optimization.

arXiv:2312.02279v1 [quant-ph] 4 Dec 2023

https://arxiv.org/abs/2312.02279

A Survey of Quantum Alternatives to Randomized Algorithms: Monte Carlo Integration and Beyond

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(Dated: March 10, 2023)

Monte Carlo sampling is a powerful toolbox of algorithmic techniques widely used for a number of applications wherein some noisy quantity, or summary statistic thereof, is sought to be estimated. In this paper, we survey the literature for implementing Monte Carlo procedures using quantum circuits, focusing on the potential to obtain a quantum advantage in the computational speed of these procedures. We revisit the quantum algorithms that could replace classical Monte Carlo and then consider both the existing quantum algorithms and the potential quantum realizations that include adaptive enhancements as alternatives to the classical procedure.

arXiv:2303.04945v1 [quant-ph] 8 Mar 2023

I. INTRODUCTION

Quantum computing promises to solve instances of certain problems currently intractable with (even high-performance) classical computers. The range of applications is vast; to name a few prominent ones, see the surveys [1], [2], and [3] discussing applications in chemistry, pharmaceuticals, and financial services, among other domains.

Monte Carlo sampling (see, for example, [4]) is a set of techniques that randomly generate numerical quantities for the purpose of simulating a statistical distribution or computing a moment or other expectation thereof (e.g., mean, variance). It is prominent in many disciplines, including computational finance [5], computational physics [6], artificial intelligence [7, 8], and various branches of engineering [9]. Although the concepts and ideas discussed in this paper readily generalize to other disciplines, we present our exposition with a focus on computational finance.

Significant computational resources are deployed for the asset pricing of, e.g., stocks, bonds, futures, and other exotic commodities such as derivatives, along with the risk management of portfolios comprising those assets. The dynamics of financial assets are subject to significant randomness, and there are several stochastic methods for fair pricing, most prominently the Black-Scholes-Merton model [10, 11]. However, machine learning techniques have become more prevalent since the financial crisis of 2008 and the subsequent recession. Classical and quasi-Monte Carlo methods are routinely used to perform computations involving random quantities [12], and feature prominently in financial pricing and risk models. See, for example, [13–15] for work on option pricing and [16] for work on credit risk assessment. See also [17]. In fact, the use of Monte Carlo methods is mandated by ever more stringent regulations in most developed countries, leading to increasing computational efforts being expended on Monte Carlo in these applications. Consequently, there is a significant interest in improving the quality and efficiency of these methods.

Given the contemporary explosion in research and development in quantum computing, there has been much recent interest in exploring quantum alternatives to classical Monte Carlo. Leading financial institutions, including HSBC [18], Barclays [19], Fidelity Investments [20], Goldman Sachs [15, 21, 22], JPMorgan Chase [14], and Mitsubishi UFJ [23], BBVA [24], actively publish research in the field, while it is likely that there will be even more industrial research that is unpublished.

The motivation for the search for quantum alternatives is rooted in the nature of these procedures, which are general and flexible enough to be effective for a wide array of possible real-life probability distributions but, in so doing, typically require a large quantity of samples to achieve good approximations. Current algorithms for particularly complex financial instruments, therefore, typically demand high-performance computing (HPC), or using a number of computing nodes in parallel to increase the number of samples while maintaining reasonable wall-clock times. However, HPC only partially mitigates the significant drawback of classical Monte Carlo, which can be expressed as slow *mixing time*. The mixing time can be thought of as a measure of how long it takes for the estimates to reach an acceptable distance from the theoretically desired quantity. In practical applications, apart from situations where simple distributions are in use, the procedure is known to mix slowly, requiring significant computing hours and thus time as well as energy expenditure.

Once fully scalable fault-tolerant error corrected quantum computers are available, quantum alternatives to Monte Carlo can potentially achieve a competitive ad-

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https://arxiv.org/pdf/2303.04945.pdf

Quantum Computing

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3. Organization of the Course
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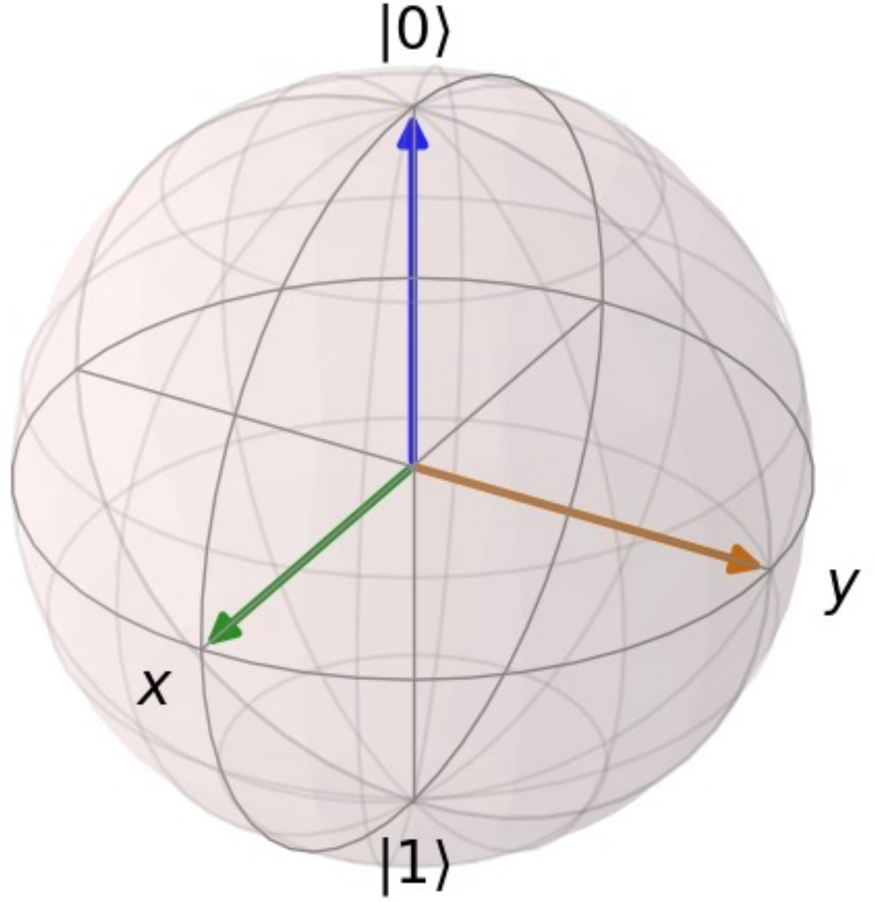
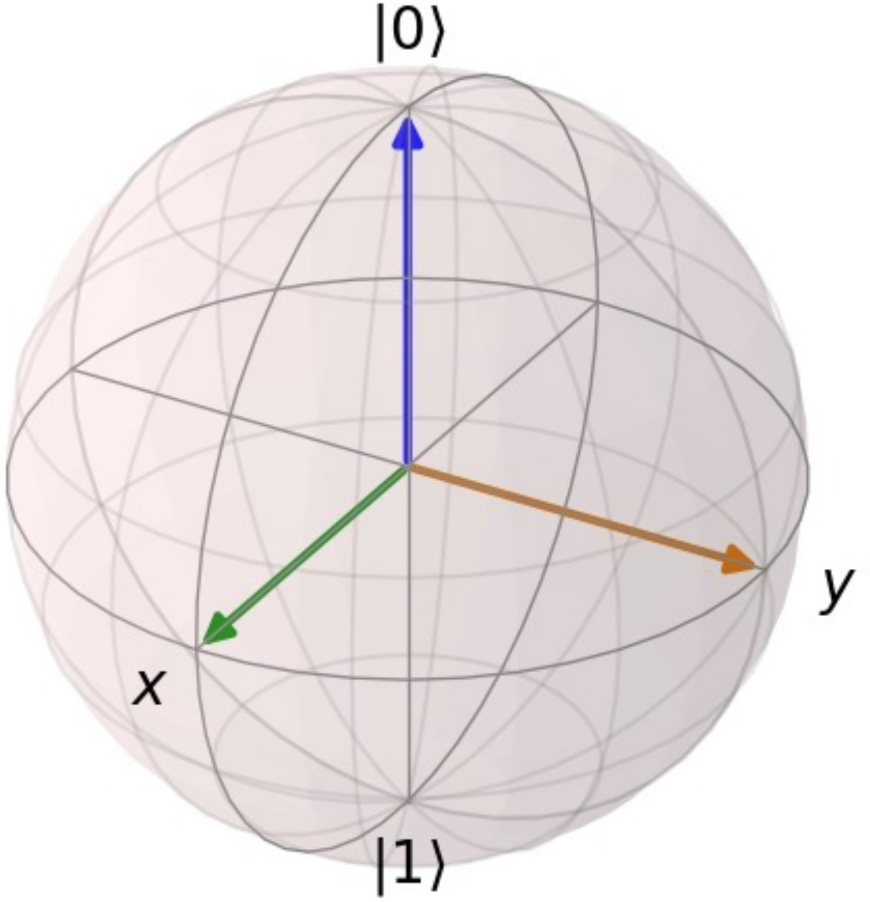
Quantum States and Qubits

- Digital computers vs. "analog computers"
- $\{0, 1\}$ vs. the state vector $|\psi\rangle = c_1 |0\rangle + c_2 |1\rangle$ of 2 complex numbers c_1, c_2
- Bloch-sphere representation thereof
- n qubits, 2^n complex numbers



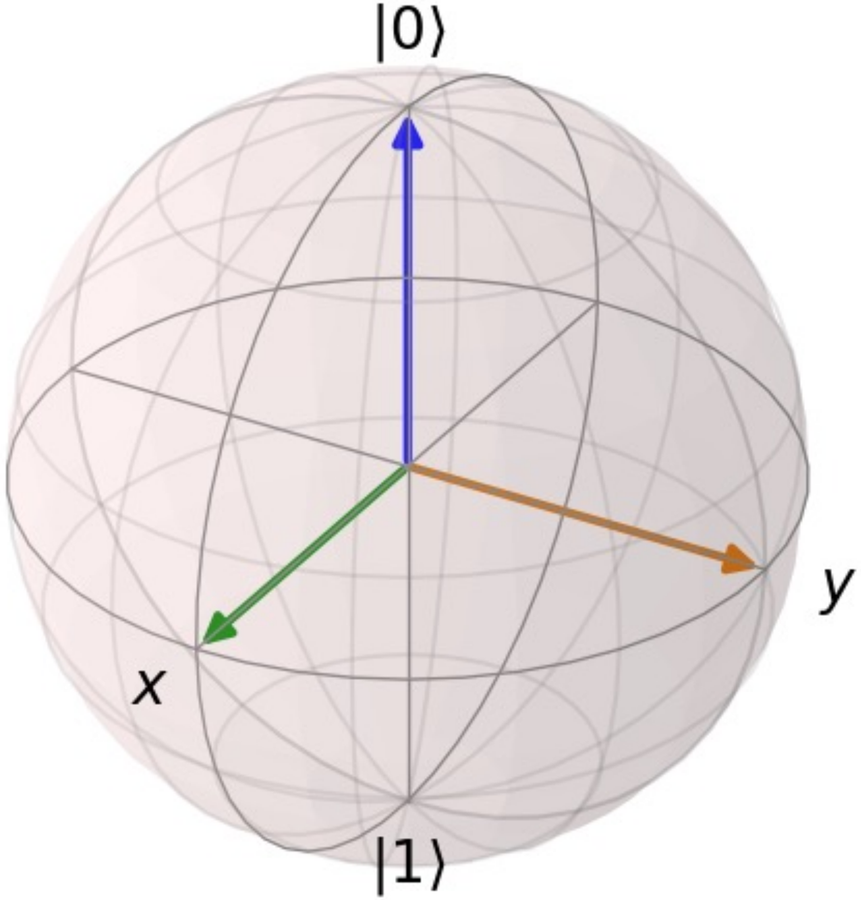
Quantum States and Qubits

- Quantum states are vectors in a complex vector space.
- A state is represented by the ket $|\psi\rangle$.
- The elements of the dual space are called bras and denoted $\langle\varphi|$.
- The inner product, or bracket, $\langle\varphi|\psi\rangle$, is a complex number, and its complex conjugate is given by $(\langle\varphi|\psi\rangle)^* = \langle\psi|\varphi\rangle$.
- We normalize the states such that $\langle\varphi|\psi\rangle=1$
- Quantum states can be in a superposition of states, $|\psi\rangle = \alpha\,|\psi_1\rangle + \beta\,|\psi_2\rangle$, for some complex numbers α, β.
- More generally, we can express any quantum state in a vector space as a superposition of the basis vectors of that vector space, $|\psi\rangle = c_1\,|a_1\rangle + c_2\,|a_3\rangle$ …, for some complex numbers c_i and basis vectors $|a_i\rangle$.



Quantum Postulates

- States are described by unit vectors in a complex vector space, and observables are described by linear Hermitian operators.
- The possible outcomes of a measurement are given by the eigenvalues of the operator corresponding to the observable being measured.
- If the system is in a state $|\psi>$, and we measure an observable A with eigenvectors $|a_j>$ and eigenvalues a_j, the probability of measuring eigenvalue a_j is given by $P(a_j) = |<a_j|\psi>|^2 = <\psi|a_j><a_j|\psi>$.
- The evolution of a quantum system is described by unitary operators.



Qubits and How to Implement Them

- Most quantum computers so far look like this:

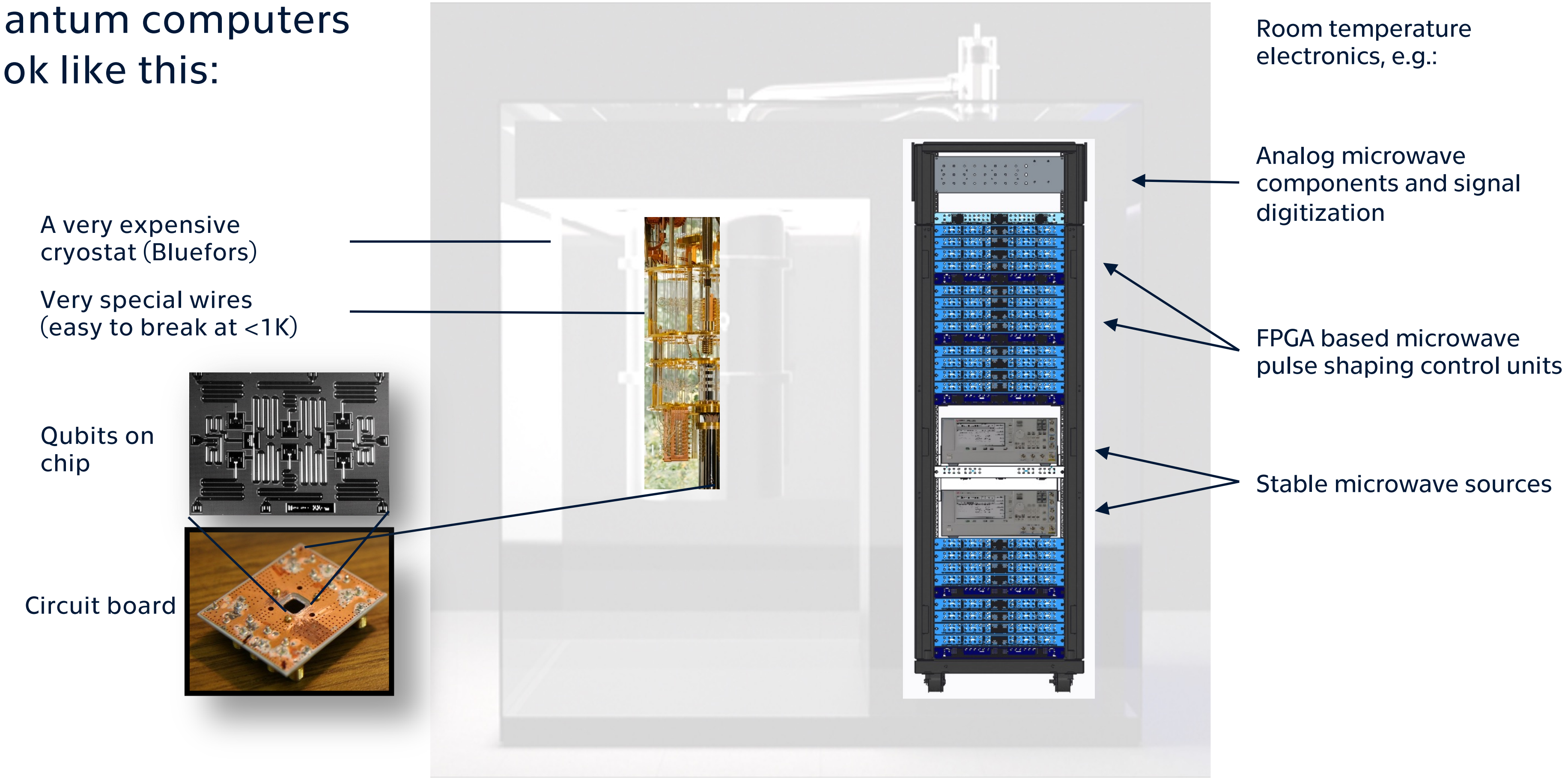


Image credit: IBM

Are we There yet?

DiVicenzo's criteria:

- A scalable physical system with well-characterized qubit
- The ability to initialize the state of the qubits to a simple fiducial state
- Long relevant decoherence times
- A "universal" set of quantum gates
- A qubit-specific measurement capability

Fortschr. Phys. **48** (2000) 9–11, 771–783

The Physical Implementation of Quantum Computation

DAVID P. DIVINCENZO

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Abstract

After a brief introduction to the principles and promise of quantum information processing, the requirements for the physical implementation of quantum computation are discussed. These five requirements, plus two relating to the communication of quantum information, are extensively explored and related to the many schemes in atomic physics, quantum optics, nuclear and electron magnetic resonance spectroscopy, superconducting electronics, and quantum-dot physics, for achieving quantum computing.

Qubits and How to Implement Them

- "What is on the chip" differs
- Superconducting qubits (transmon, ...)
- Double quantum dots (in Si, Ge, ...)
- Photonic qubits
- Ions and neutral atoms
- Fullerenes, carbon nanotubes, etc.

Physical support	Name	Information support	$\|0\rangle$	$\|1\rangle$
Photon	Polarization encoding	Polarization of light	Horizontal	Vertical
	Number of photons	Fock state	Vacuum	Single photon state
	Time-bin encoding	Time of arrival	Early	Late
Coherent state of light	Squeezed light	Quadrature	Amplitude-squeezed state	Phase-squeezed state
Electrons	Electronic spin	Spin	Up	Down
	Electron number	Charge	No electron	One electron
Nucleus	Nuclear spin addressed through NMR	Spin	Up	Down
Optical lattices	Atomic spin	Spin	Up	Down
Josephson junction	Superconducting charge qubit	Charge	Uncharged superconducting island (Q=0)	Charged superconducting island (Q=2e, one extra Cooper pair)
	Superconducting flux qubit	Current	Clockwise current	Counterclockwise current
	Superconducting phase qubit	Energy	Ground state	First excited state
Singly charged quantum dot pair	Electron localization	Charge	Electron on left dot	Electron on right dot
Quantum dot	Dot spin	Spin	Down	Up
Gapped topological system	Non-abelian anyons	Braiding of Excitations	Depends on specific topological system	Depends on specific topological system
Vibrational qubit[10]	Vibrational states	Phonon/vibron	$\|01\rangle$ superposition	$\|10\rangle$ superposition
van der Waals heterostructure[11]	Electron localization	Charge	Electron on bottom sheet	Electron on top sheet

https://en.wikipedia.org/wiki/Qubit

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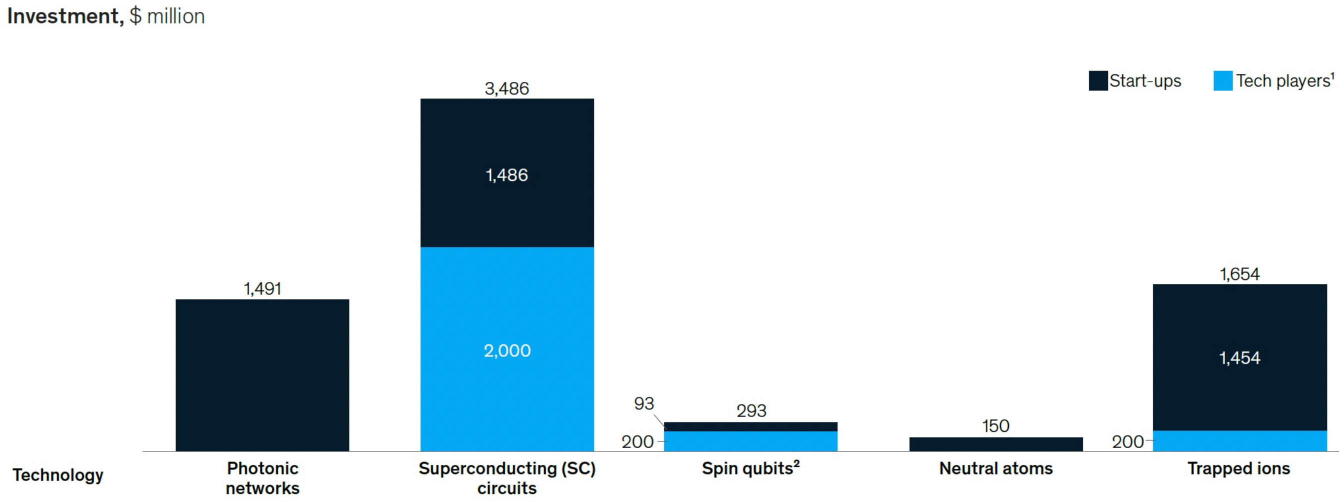
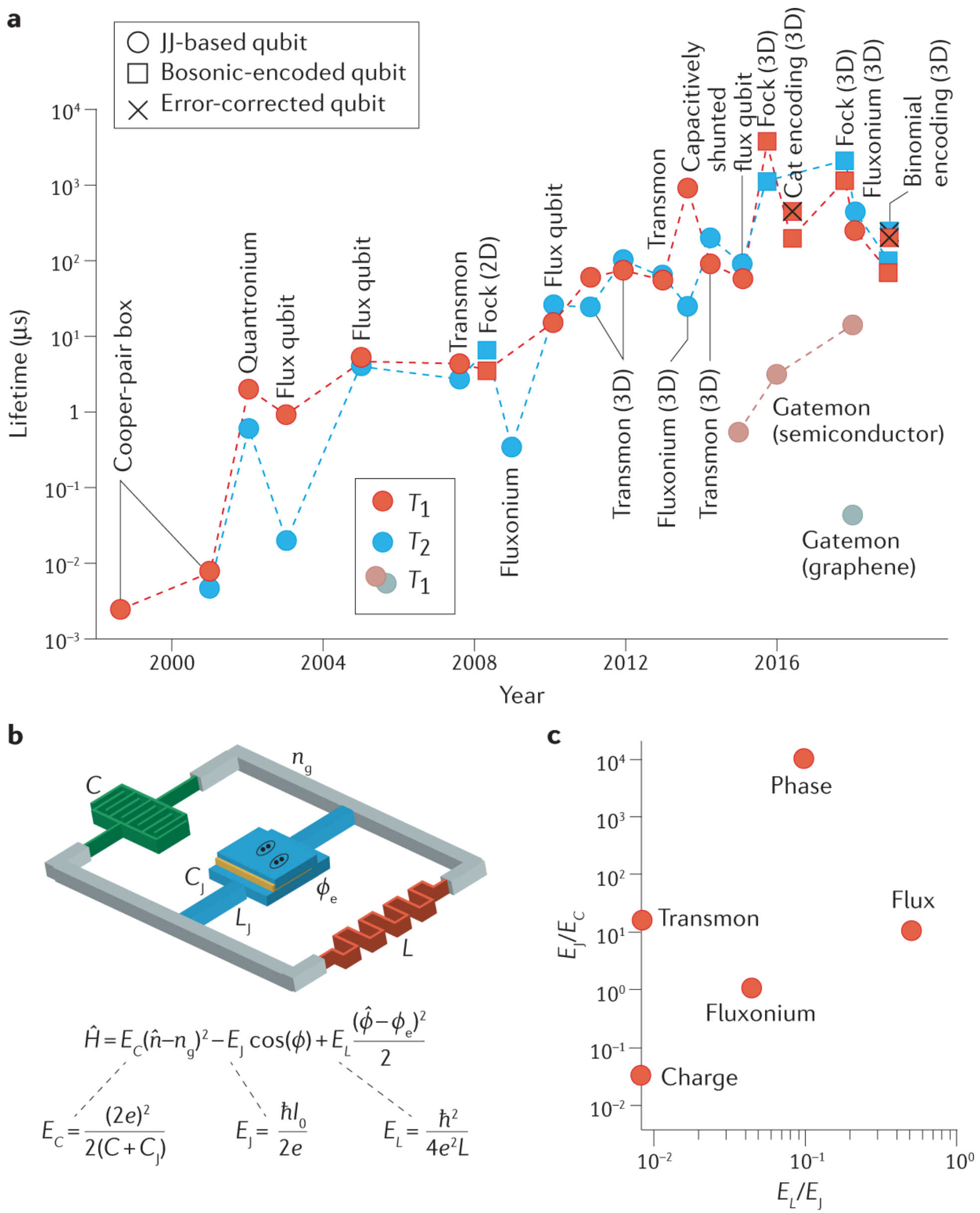


[1]Assumptions: $500 million per major player (Google, IBM, Alibaba, AWS), $200 million per medium player (Honeywell before merger with CQC into Quantinuum, Intel).
[2]We examine electron spins in silicon quantum dots, as other spin qubits are generally not considered for applications in quantum computing, eg, NV centers in diamond are unlikely to be a good qubit for computing; however, they can serve as quantum sensors.

https://www.mckinsey.com/capabilities/mckinsey-digital/our-insights/quantum-technology-sees-record-investments-progress-on-talent-gap#/

Qubits and How to Implement Them



- 1962: Josephson effect
  tunneling of superconducting Cooper pairs
  (Nobel Prize in Physics, 1973)
- Based on Josephson junction, superconducting qubits ess. implement a quantum oscillator

- Transmon qubits @ IBM
- Xmon @ Google
- Cca. At 10 mK

https://www.nature.com/articles/s41578-021-00370-4

Qubits and How to Implement Them

- 1963: Quantum well
  with discrete energy values
  (Kroemer, Alferov, Kazarinov)
- Double quatum dots @ Intel, ...
- At 1K at Intel (?), up to 20 K (Myronov)

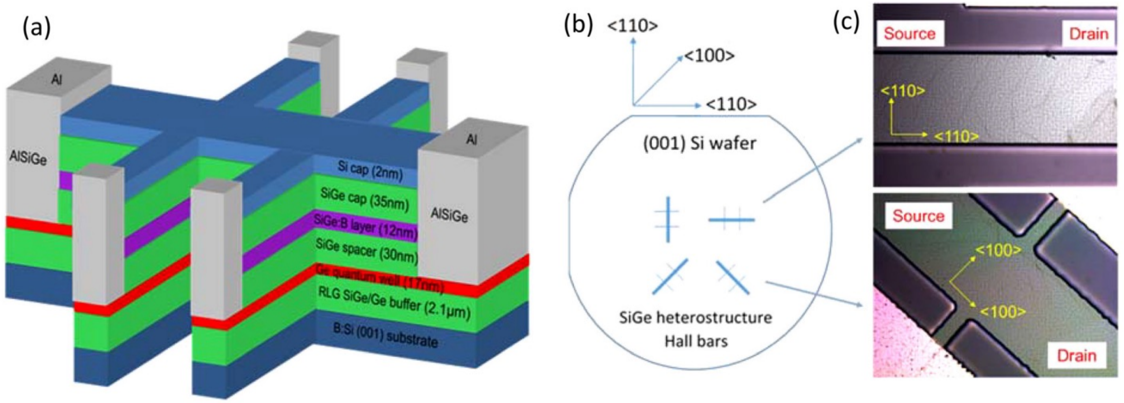
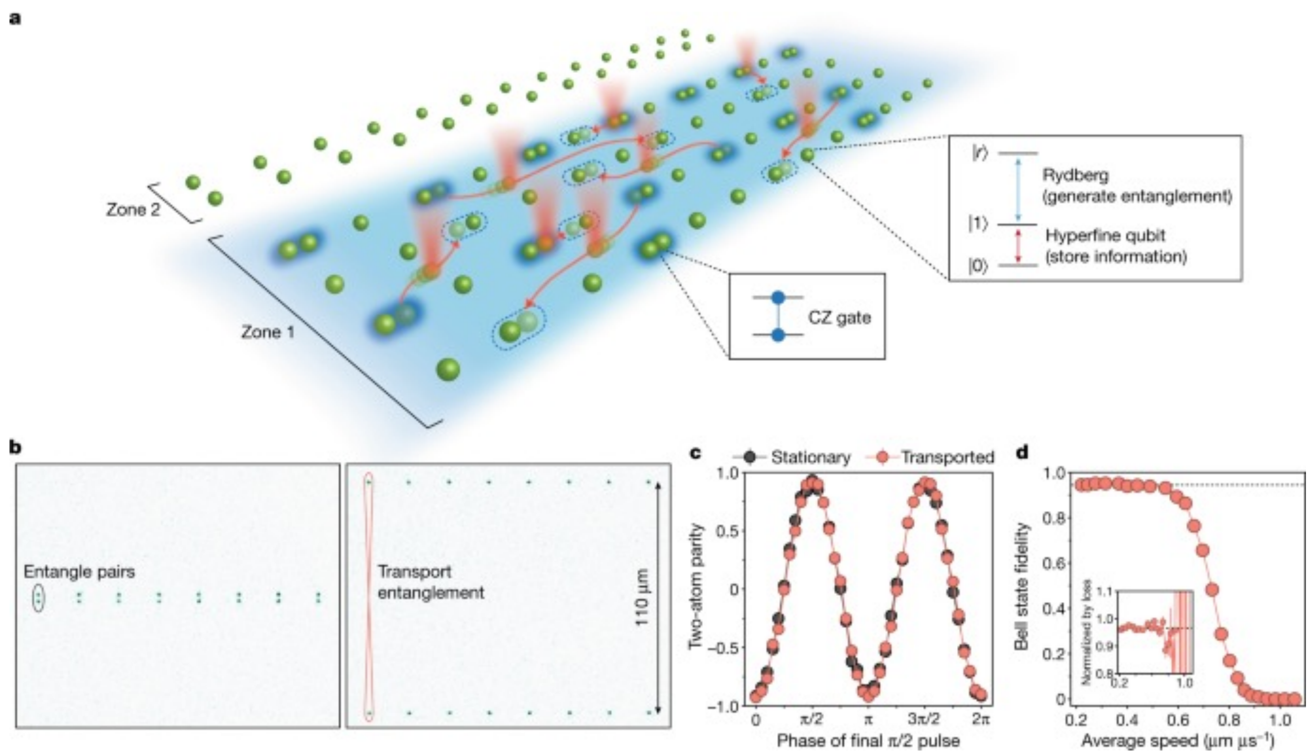


FIG. 1. (a) Schematic of the Hall bar device used, showing the composition of the heterostructure. (b) (001) plane of the wafer, illustrating the ⟨110⟩ and ⟨100⟩ directions. (c) Optical images of the Ge heterostructure Hall bars showing cross hatching from epitaxial growth. This pattern is aligned to the ⟨110⟩ directions.

Characteristics	Holes in strained Ge	Electrons in Si
Effective mass (m_0)	0.035	0.19 m
Coherence time (T2*)	150 μs	120 μs community accepts 20 μs
Rabi frequency	140 MHz	10 MHz
Single-qubit operation fidelity	99.3 %	99.9 %

https://aip.scitation.org/doi/full/10.1063/1.5010933

Qubits and How to Implement Them



- Neutral atoms @ QuEra / Amazon / Harvard / …
- 2D optical tweezer array
- Cca. at 25 μK (!)
- Entangled atoms cca. 110 μm apart

- Ions @ IoniQ / Alpine Quantum / Innsbruck / …

https://www.nature.com/articles/s41586-022-04592-6

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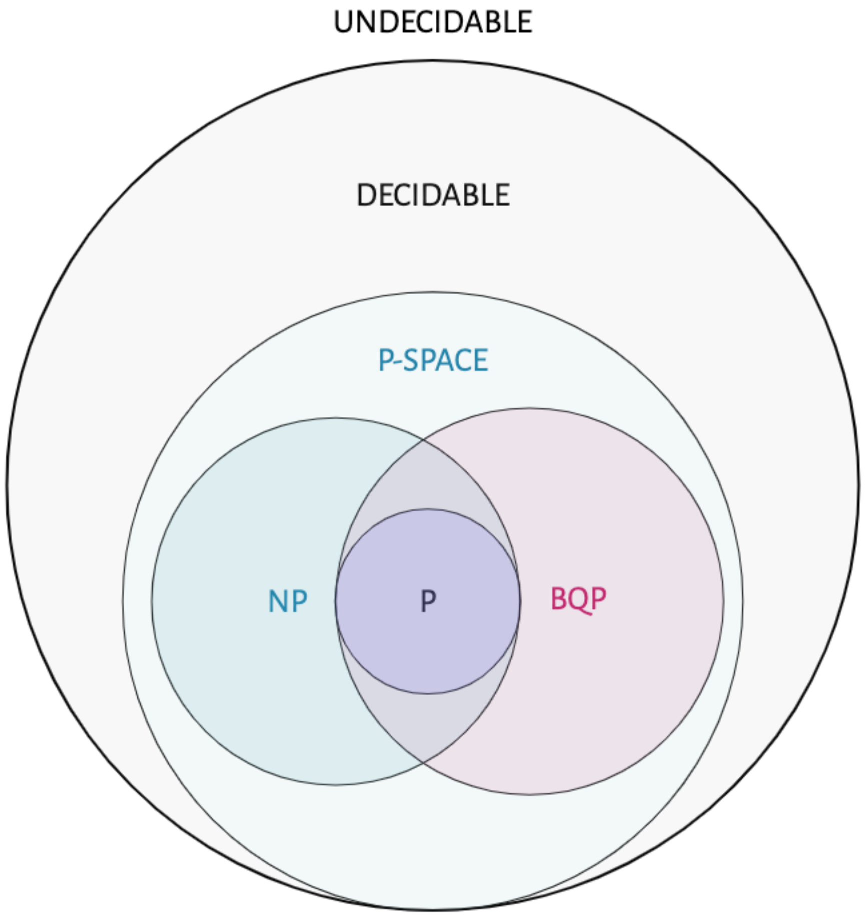
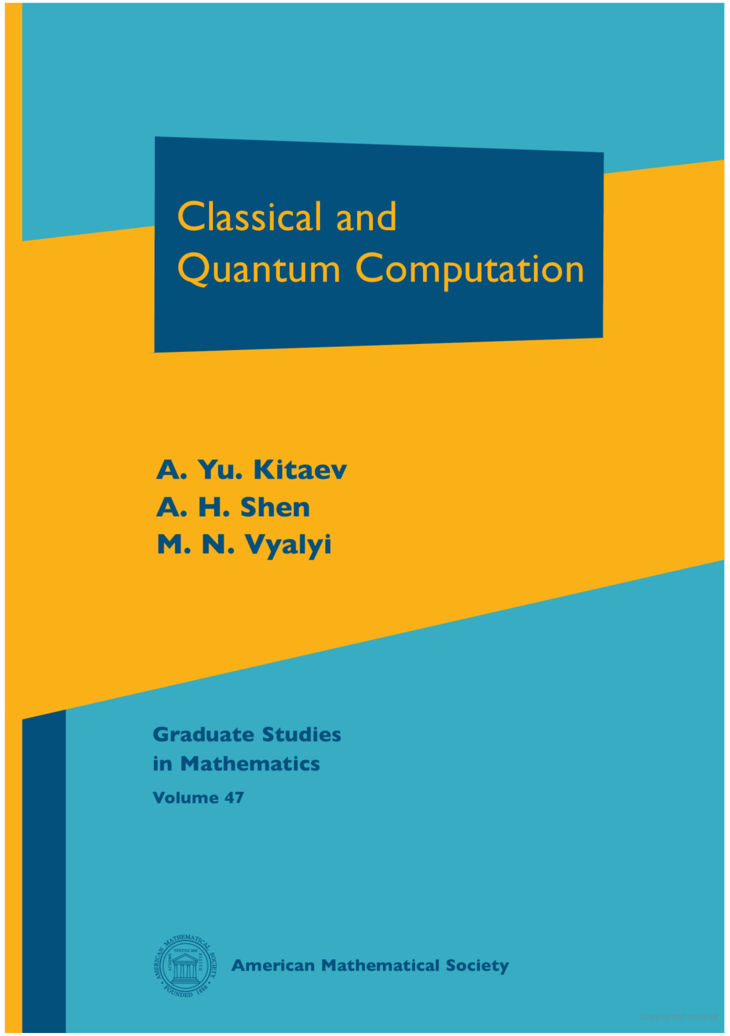
Computational Complexity

- P: a class of problems with certificates computable by a Turing machine in polynomial time. E.g. shortest path in a graph.
- NP: a class of problems with certificates verifiable by a Turing machine in polynomial time. E.g. the travelling salesman problem.
- BPP: a classical class of randomized algorithms.
- BQP: a “quantum equivalent” class to BPP.
- BQNP = QMA (Quantum Merlin Arthur): a "quantum equivalent" to NP. Specifically: A class of problems with polynomial-size quantum proof (a quantum state) that convinces a polynomial time quantum verifier (running on a quantum computer) with high probability.
- BQNP = QMA includes NP. It is not clear whether this is strict.



The zoo of classical and quantum complexity classes under the common assumptions that NP ! =\P and NP != BQP. Image credit: Jakub Marecek and Georgios Korpas.

Computational Complexity



14.4. Local Hamiltonian is BQNP-complete.

Theorem 14.3. *The problem* LOCAL HAMILTONIAN *is* BQNP-*complete with respect to the Karp reduction.*

The rest of this section constitutes a proof of this theorem. The main idea goes back to Feynman [**24**]: replacing a unitary evolution by a time independent Hamiltonian (i.e., transition from the circuit to a local Hamiltonian).

Thus, suppose we have a circuit $U = U_L \cdots U_1$ of size L. We will assume that U acts on N qubits, the first m of which initially contain Merlin's message $|\xi\rangle$, the rest being initialized by 0. The gates U_j act on pairs of qubits.

14.4.1. ***The Hamiltonian associated with the circuit.*** It acts on the space

$$\mathcal{L} = \mathcal{B}^{\otimes N} \otimes \mathbb{C}^{L+1},$$

where the first factor is the space on which the circuit acts, whereas the second factor is the space of a step counter (clock). The Hamiltonian consists of three terms which will be defined later,

$$H = H_{\text{in}} + H_{\text{prop}} + H_{\text{out}}.$$

We are interested in the minimum eigenvalue of this Hamiltonian, or the minimum of the *cost function* $f(|\eta\rangle) = \langle\eta|H|\eta\rangle$ over all vectors $|\eta\rangle$ of unit length. We will try to arrange that the Hamiltonian has a small eigenvalue if and only if there exists a quantum state $|\xi\rangle \in \mathcal{B}^{\otimes m}$ causing U to output 1 with high probability. In such a case, the minimizing vector $|\eta\rangle$ will be related to that $|\xi\rangle$ in the following way:

$$|\eta\rangle = \frac{1}{\sqrt{L+1}} \sum_{j=0}^{L} U_j \cdots U_1 |\xi, 0\rangle \otimes |j\rangle.$$

In constructing the terms of the Hamiltonian, we will try to "enforce" this structure of the vector $|\eta\rangle$ by imposing "penalties" that increase the cost function whenever $|\eta\rangle$ deviates from the indicated form.

The term H_{in} corresponds to the condition that, at step 0, all the qubits but m are in state $|0\rangle$. Specifically,

$$H_{\text{in}} = \left(\sum_{s=m+1}^{N} \Pi_s^{(1)} \right) \otimes |0\rangle\langle 0|, \qquad (14.4)$$

where $\Pi_s^{(\alpha)}$ is the projection onto the subspace of vectors for which the s-th qubit equals α. The second factor in this formula acts on the space of the counter. (Informally speaking, the term $\Pi_s^{(1)} \otimes |0\rangle\langle 0|$ "collects a penalty" by

https://www.ams.org/books/gsm/047/

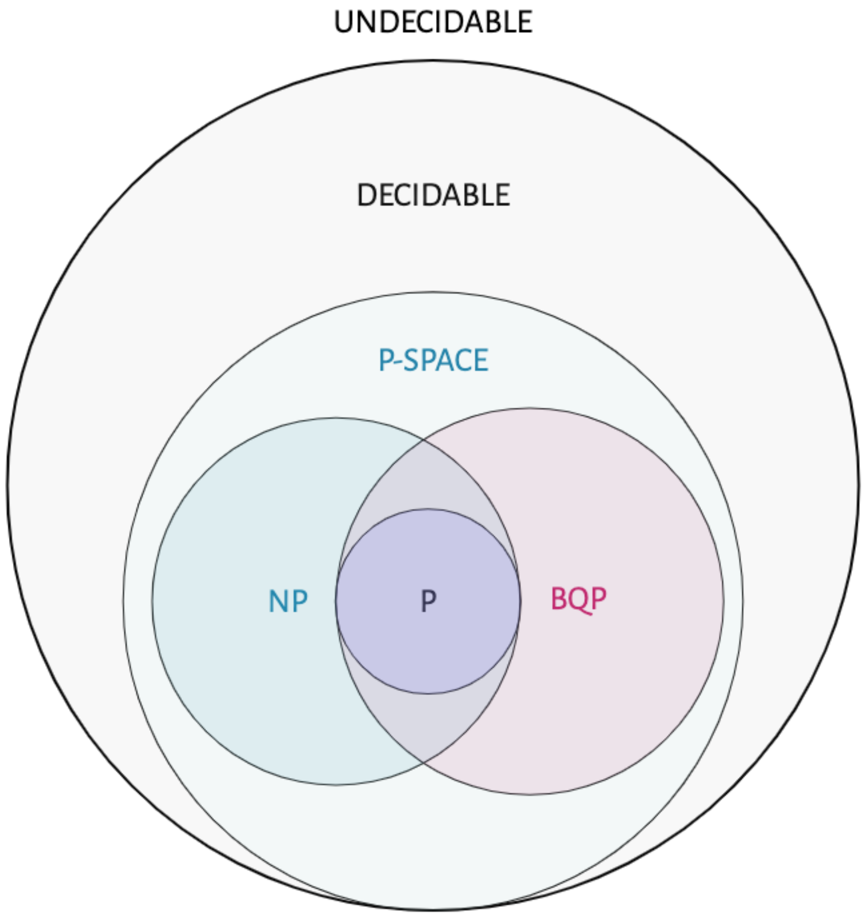
Computational Complexity

Let us consider a different class of problems, related to counting satisfying assignments, numerical integration, etc (#P):

- Classical Monte Carlo with N sample paths achieves error $O(1/\sqrt{N})$
- Quasi Monte Carlo methods on classical computers w/ error $O(\log(N)^s/N)$ for some s that may depend on dimension.
- Quantum replacements of Monte Carlo achieve error $O(1/N)$

This is often mis-understood in the hunt for elusive algorithms for NP-Complete problems!

Even $P^{\#P}$ is within PSPACE.



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Three Use Cases

Cryptography

- The Big Scare
- Quantum Cryptography
- Post-quantum Cryptography

Simulation

- Monte Carlo Replacements

Optimization & Control

- Variational Algorithms?

arXiv > quant-ph > arXiv:2006.14510

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Quantum Physics

[Submitted on 25 Jun 2020 (v1), last revised 28 Jan 2021 (this version, v3)]

Quantum Computing for Finance: State of the Art and Future Prospects

Daniel J. Egger, Claudio Gambella, Jakub Marecek, Scott McFaddin, Martin Mevissen, Rudy Raymond, Andrea Simonetto, Stefan Woerner, Elena Yndurain

This article outlines our point of view regarding the applicability, state-of-the-art, and potential of quantum computing for problems in finance. We provide an introduction to quantum computing as well as a survey on problem classes in finance that are computationally challenging classically and for which quantum computing algorithms are promising. In the main part, we describe in detail quantum algorithms for specific applications arising in financial services, such as those involving simulation, optimization, and machine learning problems. In addition, we include demonstrations of quantum algorithms on IBM Quantum back-ends and discuss the potential benefits of quantum algorithms for problems in financial services. We conclude with a summary of technical challenges and future prospects.

Comments: 24 pages
Subjects: **Quantum Physics (quant-ph)**; Statistical Finance (q-fin.ST)
Cite as: arXiv:2006.14510 **[quant-ph]**
(or arXiv:2006.14510v3 **[quant-ph]** for this version)
https://doi.org/10.48550/arXiv.2006.14510
Journal reference: IEEE Transactions on Quantum Engineering, vol. 1, pp. 1-24, 2020, Art no. 3101724
Related DOI: https://doi.org/10.1109/TQE.2020.3030314

The Big Scare

⟨Quantum
the open journal for quantum science

PAPERS PERSPECTIVES

How to factor 2048 bit RSA integers in 8 hours using 20 million noisy qubits

Craig Gidney[1] and Martin Ekerå[2,3]

[1]Google Inc., Santa Barbara, California 93117, USA
[2]KTH Royal Institute of Technology, SE-100 44 Stockholm, Sweden
[3]Swedish NCSA, Swedish Armed Forces, SE-107 85 Stockholm, Sweden

Featured in Physics Editors' Suggestion

Factoring 2048-bit RSA Integers in 177 Days with 13 436 Qubits and a Multimode Memory

Élie Gouzien and Nicolas Sangouard
Phys. Rev. Lett. **127**, 140503 – Published 28 September 2021

Physics See synopsis: Far Fewer Qubits Required for "Quantum Memory" Quantum Computers

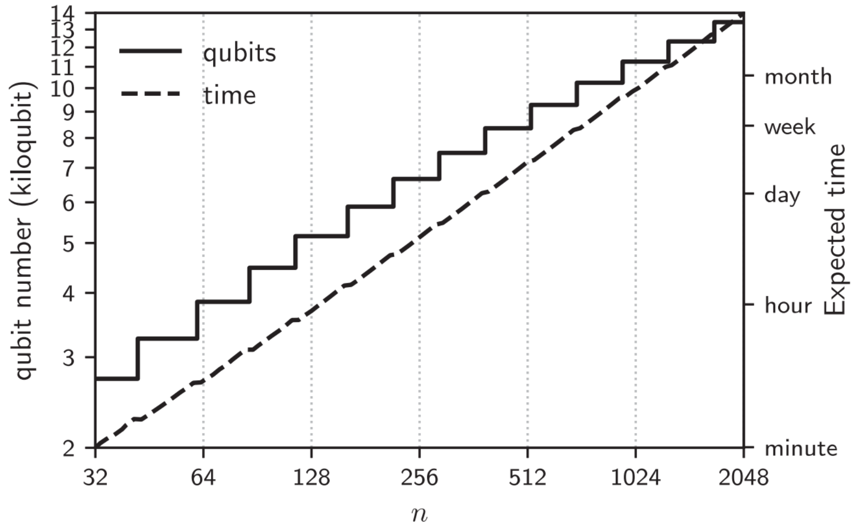


FIG. 2. Number of qubits in the processor and run-time to factor n-bit RSA integers with a computer architecture using a multimode memory.

Quantum Cryptography

Long Distance QKD System

The Long Distance QKD System operates with a quantum channel in the telecom C-band for the longest possible range and highest possible secure key rate. It can tolerate limited bandwidths of multiplexed data within the C-band.

Key Features:

1. Typical key rate = 300 kb/s for 10dB loss
2. Range of up to 120km
3. Two fibers required
4. Efficient BB84 protocol with decoy states and phase encoding
5. Key failure probability of less than 10^{-10} equivalent to less than once in 30,000 years
6. Proprietary self-differencing semiconductor detectors

Why Toshiba QKD

We started research into quantum cryptography in 2003 at the Cambridge Research Laboratory of Toshiba Research Europe Limited. Since then we have demonstrated a number of notable world firsts. We were the first to announce quantum key distribution over 100 km of fiber in 2004 and the first with a continuous key rate exceeding 1 Mbit/second in 2010 and 10 Mbit/second in 2017.

Post-Quantum Cryptography

NIST

Information Technology Laboratory

COMPUTER SECURITY RESOURCE CENTER

Check for updates

NIST IR 8413-upd1

Status Report on the Third Round of the NIST Post-Quantum Cryptography Standardization Process

PROJECTS POST-QUANTUM CRYPTOGRAPHY

Post-Quantum Cryptography PQC

Selected Algorithms 2022

Selected Algorithms: Public-key Encryption and Key-establishment Algorithms

Algorithm	Algorithm Information	Submitters	Comments
CRYSTALS-KYBER	Zip File (7MB) IP Statements Website	Peter Schwabe Roberto Avanzi Joppe Bos Leo Ducas Eike Kiltz Tancrede Lepoint Vadim Lyubashevsky John M. Schanck	Submit Comment View Comments

Post-Quantum Cryptography

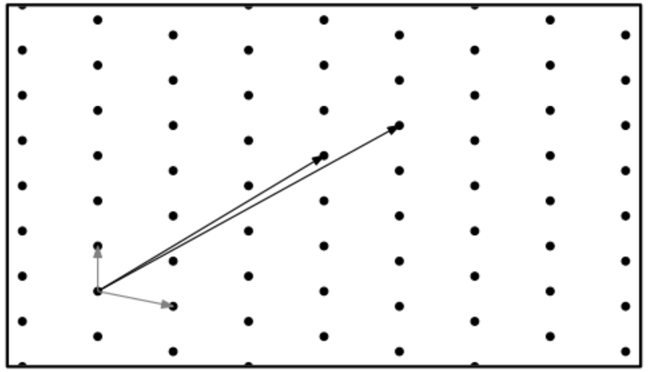
Kyber is based on lattice cryptography, which are NP-Hard and not known to be PSPACE-Hard. Non-trivially, the problem is related to SIVP: Given a lattice basis, find k linearly independent lattice vectors minimizing the maximum of their norms.

Worst-case to average-case reductions for module lattices

Adeline Langlois & Damien Stehlé

Designs, Codes and Cryptography **75**, 565–599 (2015) | Cite this article

1912 Accesses | **171** Citations | **3** Altmetric | Metrics



Kyber is already baing used:

- Cloudflare Interoperable, Reusable Cryptographic Library
- Amazon Web Services Key Management Service
- IBM's World's First Quantum Computing Safe Tape Drive (using Kyber and Dilithium).

Monte Carlo Replacements

Much of what banks do, boils down to Monte Carlo:

- Risk assessment and mandated by regulators
- Internal risk assessment
- Pricing of a variety of products (e.g. credit, European call options).

What error do I get with N sample paths?

- Classical Monte Carlo methods $O(1/\sqrt{N})$
- Quasi-Monte-Carlo methods $O(\log(N)^s/N)$
- Quantum replacements $O(1/N^2)$

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Credit Risk Analysis Using Quantum Computers

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Option Pricing using Quantum Computers

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A Threshold for Quantum Advantage in Derivative Pricing

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Optimization & Monte Carlo Replacements

Quantum Optimization: Potential, Challenges, and the Path Forward*

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Recent advances in quantum computers are demonstrating the ability to solve problems at a scale beyond brute force classical simulation. As such, a widespread interest in quantum algorithms has developed in many areas, with optimization being one of the most pronounced domains. Across computer science and physics, there are a number of algorithmic approaches, often with little linkage. This is further complicated by the fragmented nature of the field of mathematical optimization, where major classes of optimization problems, such as combinatorial optimization, convex optimization, non-convex optimization, and stochastic extensions, have devoted communities. With these aspects in mind, this work draws on multiple approaches to study quantum optimization. Provably exact versus heuristic settings are first explained using computational complexity theory — highlighting where quantum advantage is possible in each context. Then, the core building blocks for quantum optimization algorithms are outlined to subsequently define prominent problem classes and identify key open questions that, if answered, will advance the field. The effects of scaling relevant problems on noisy quantum devices are also outlined in detail, alongside meaningful benchmarking problems. We underscore the importance of benchmarking by proposing clear metrics to conduct appropriate comparisons with classical optimization techniques. Lastly, we highlight two domains – finance and sustainability – as rich sources of optimization problems that could be used to benchmark, and eventually validate, the potential real-world impact of quantum optimization.

https://arxiv.org/abs/2312.02279

A Survey of Quantum Alternatives to Randomized Algorithms: Monte Carlo Integration and Beyond

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Monte Carlo sampling is a powerful toolbox of algorithmic techniques widely used for a number of applications wherein some noisy quantity, or summary statistic thereof, is sought to be estimated. In this paper, we survey the literature for implementing Monte Carlo procedures using quantum circuits, focusing on the potential to obtain a quantum advantage in the computational speed of these procedures. We revisit the quantum algorithms that could replace classical Monte Carlo and then consider both the existing quantum algorithms and the potential quantum realizations that include adaptive enhancements as alternatives to the classical procedure.

I. INTRODUCTION

Quantum computing promises to solve instances of certain problems currently intractable with (even high-performance) classical computers. The range of applications is vast; to name a few prominent ones, see the surveys [1], [2], and [3] discussing applications in chemistry, pharmaceuticals, and financial services, among other domains.

Monte Carlo sampling (see, for example, [4]) is a set of techniques that randomly generate numerical quantities for the purpose of simulating a statistical distribution or computing a moment or other expectation thereof (e.g., mean, variance). It is prominent in many disciplines, including computational finance [5], computational physics [6], artificial intelligence [7, 8], and various branches of engineering [9]. Although the concepts and ideas discussed in this paper readily generalize to other disciplines, we present our exposition with a focus on computational finance.

Significant computational resources are deployed for the asset pricing of, e.g., stocks, bonds, futures, and other exotic commodities such as derivatives, along with the risk management of portfolios comprising those assets. The dynamics of financial assets are subject to significant randomness, and there are several stochastic methods for fair pricing, most prominently the Black-Scholes-Merton model [10, 11]. However, machine learning techniques have become more prevalent since the financial crisis of 2008 and the subsequent recession. Classical and quasi-Monte Carlo methods are routinely used to perform computations involving random quantities [12], and feature prominently in financial pricing and risk models. See, for example, [13–15] for work on option pricing and [16] for work on credit risk assessment. See also [17]. In fact, the use of Monte Carlo methods is mandated by ever more stringent regulations in most developed countries, leading to increasing computational efforts being expended on Monte Carlo in these applications. Consequently, there is a significant interest in improving the quality and efficiency of these methods.

Given the contemporary explosion in research and development in quantum computing, there has been much recent interest in exploring quantum alternatives to classical Monte Carlo. Leading financial institutions, including HSBC [18], Barclays [19], Fidelity Investments [20], Goldman Sachs [15, 21, 22], JPMorgan Chase [14], and Mitsubishi UFJ [23], BBVA [24], actively publish research in the field, while it is likely that there will be even more industrial research that is unpublished.

The motivation for the search for quantum alternatives is rooted in the nature of these procedures, which are general and flexible enough to be effective for a wide array of possible real-life probability distributions but, in so doing, typically require a large quantity of samples to achieve good approximations. Current algorithms for particularly complex financial instruments, therefore, typically demand high-performance computing (HPC), or using a number of computing nodes in parallel to increase the number of samples while maintaining reasonable wall-clock times. However, HPC only partially mitigates the significant drawback of classical Monte Carlo, which can be expressed as slow *mixing time*. The mixing time can be thought of as a measure of how long it takes for the estimates to reach an acceptable distance from the theoretically desired quantity. In practical applications, apart from situations where simple distributions are in use, the procedure is known to mix slowly, requiring significant computing hours and thus time as well as energy expenditure.

Once fully scalable fault-tolerant error corrected quantum computers are available, quantum alternatives to Monte Carlo can potentially achieve a competitive ad-

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https://arxiv.org/pdf/2303.04945.pdf

Quantum Computing

1. Motivation: “A social phenomenon”
2. Motivation: Opportunities and Limitations
3. Organization of the Course
4. Qubits and How to Implement them
5. A Theoretical Computer Science Point of View
6. Three Use Cases in Financial Services

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