Quantum Computing

Exercises 1: Intro to Quantum Physics

1. *a) Show that the 'in' and 'out' states defined as:*

$$|i\rangle = \frac{1}{\sqrt{2}}(|u\rangle + i|d\rangle)$$
$$|o\rangle = \frac{1}{\sqrt{2}}(|u\rangle - i|d\rangle)$$

are orthogonal.
b) Calculate the expectation values of σ_y in the states $|u\rangle$ and $|i\rangle$, and of σ_z in the state $|o\rangle$.

2. *a) Normalise the state*

$$|\psi\rangle = 3i|u\rangle + (1-2i)|d\rangle.$$

b) For this (normalised) state, calculate the probability of getting both positive $(+1)$ and negative (-1) spin eigenvalues by measuring σ_z.

3. *(Nielsen & Chuang Ex. 2.11 [Eigendecomposition of a Pauli matrix])*
Find the eigenvectors, eigenvalues and diagonal representations of σ_x.

4. *(Hermitian operators)*
For a hermitian matrix $\mathbf{A}$, that is, a matrix that satisfies $\mathbf{A} = \mathbf{A}^\dagger$, show that:
a) Different eigenvalues have orthogonal eigenvectors.
b) All its eigenvalues are real. Does the converse also hold, that is, if the spectrum (the set of all eigenvalues) of a matrix is in $\mathbb{R}$, is it then a hermitian matrix?

5. *(Unitary operators)*
Now, consider a unitary matrix, one for which

$$UU^\dagger = \mathbb{I} \iff U^\dagger U = \mathbb{I} \iff U^{-1} = U^\dagger$$

holds. Prove that its eigenvalues are of the form $e^{i\theta}$ and that eigenvectors of different eigenvalues must be orthogonal as well.