First homework assignment announced 15. 3. 2024, due 5. 4. 2024 (as a zip file in Brute)

Q1: Consider the 2-qubit state $|\psi^+\rangle := \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$. Calculate the expectation values in this state for the operators

a) $H \otimes H$, and **(1.5p)**

b) $H \otimes \sigma_z$. **(1.5p)**

Here H is the Hadamard operator, $H := \frac{1}{\sqrt{2}}(\sigma_x + \sigma_z)$, and σ_x, σ_z the ordinary Pauli operators.

Q2: Consider the Hamiltonian operator of a 2-dimensional quantum harmonic oscillator

$$H = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{1}{2}m\omega^2(x^2 + y^2).$$

This can be written as the sum of the Hamiltonian of two one-dimensional oscillators

$$H = H_x + H_y, \quad H_j = \frac{1}{2m}p_j^2 + \frac{1}{2}m\omega^2 j^2,$$

for $j = x, y$. The momentum and position operators satisfy the commutation relations: $[x, p_x] = [y, p_y] = i\hbar$, while the rest are zero, i.e. $[x, y] = [x, p_y] = [y, p_x] = [p_x, p_y] = 0$.

a) Does H_x and H_y commute? **(1.5p)**

b) Can you construct a non-trivial (i.e. not zero or identity) operator, using only multiples of x, y, p_x and p_y, that commutes with the full Hamiltonian H? If possible, what does this tell you about this quantity? **(1.5p)**

Q3: Alice and Bob are studying a 3-dimensional quantum system $|\psi\rangle \in \mathbb{C}^3$. Alice measures an observable that can take values red, green and blue (or r, g and b) while Bob measures an observable that gives values sweet, tangy or umami (or s, t and u). If Alice find the result r, then Bob finds that he finds the corresponding states s, t or u with probabilities 0, p and $1-p$, respectively. If on the other hand, Alice finds g, Bob's probabilities becomes q, 0 and $1-q$, for the values s, t, and u, respectively.

a) Which combinations of values are allowed for p and q? **(2p)**

b) What are Bob's probabilities if Alice finds the result b? **(2p)**