Neuroinformatics 2024

March 27, 2024

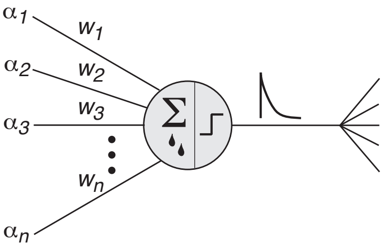
Lecture 4: Simplified neuron

The leaky integrate-and-fire neuron

$$\tau_{\mathrm{m}} \frac{\mathrm{d}v(t)}{\mathrm{d}t} = -(v(t) - E_L) + RI(t), \tag{1}$$

$$v(t^{\mathrm{f}}) = \vartheta. \tag{2}$$

$$\lim_{\delta \to 0} v(t^{\mathrm{f}} + \delta) = v_{\mathrm{res}}, \tag{3}$$



The leaky integrate-and-fire neuron - analytical solution

- Very short input current

$$
\begin{aligned}
\tau_m \frac{dv(t)}{dt} + v(t) &= 0 \\
v(t) &= \exp -\left(\frac{t}{-\tau_m}\right)
\end{aligned}
$$

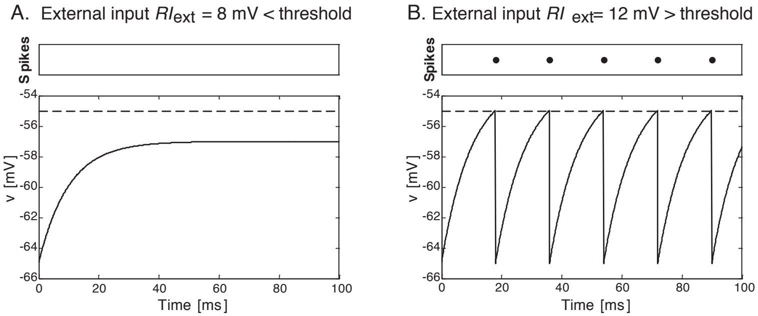
- Constant small current $RI < \theta$

$$
\begin{aligned}
\frac{dv}{dt} &= 0 \\
v &= RI \\
v(t) &= RI\left(1 - \exp -\left(\frac{t}{-\tau_m}\right) + \frac{v(t=0)}{RI} \exp -\left(\frac{t}{-\tau_m}\right)\right)
\end{aligned} \tag{4}
$$

The leaky integrate-and-fire neuron - CODE

```
%% Simulation of (leaky) integrate-and-fire neuron
clear; clf;

%% parameters of the model
dt=0.1;         % integration time step [ms]
tau=10;         % time constant [ms]
E_L=-65;        % resting potential [mV]
theta=-55;      % firing threshold [mV]
RI_ext=12;       % constant external input [mA/Ohm]

%% Integration with Euler method
t_step=0; v=E_L;
for t=0:dt:100;
    t_step=t_step+1;
    s=v>theta;
    v=s*E_L+(1-s)*(v-dt/tau*((v-E_L)-RI_ext));
    v_rec(t_step)=v;
    t_rec(t_step)=t;
    s_rec(t_step)=s;
end

%% Plotting results
subplot('position',[0.13 0.13 1-0.26 0.6])
  plot(t_rec,v_rec);
  hold on; plot([0 100],[-55 -55],'--');
  axis([0 100 -66 -54]);
  xlabel('Time [ms]'); ylabel('v [mV]')

subplot('position',[0.13 0.8 1-0.26 0.1])
  plot(t_rec,s_rec,'.','markersize',20);
  axis([0 100 0.5 1.5]);
  set(gca,'xtick',[],'ytick',[])
  ylabel('Spikes')
```

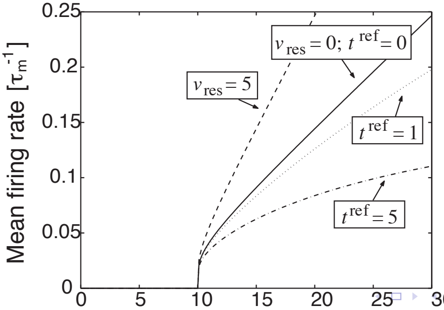
The leaky integrate-and-fire neuron (cont.)



The LIF-neuron (cont.): Gain function

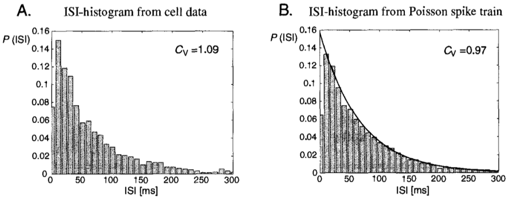
- Gain transfer activation
- The inverse of the first passage time t^f defines the **firing rate**
- Spikes occurs at $t = t^f = 0$, let's substitute $v(t = 0) = v_{\mathrm{res}}$, $v(t) = \vartheta$ into (4), t_{ref} is absolutory refractory period

$$
\begin{aligned}
t^f &= -\tau_m \ln\left(\frac{\vartheta - RI}{v_{\mathrm{res}} - RI}\right) \\
\bar{r} &= \left(t^{\mathrm{ref}} - \tau_{\mathrm{m}} \ln \frac{\vartheta - RI}{v_{\mathrm{res}} - RI}\right)^{-1}
\end{aligned}
\tag{5}
$$



Spike-time variability

- Inter-spike interval (ISI)
- IF neuron - constant ISI $C_V = 0$, A-cortical cell(Broadmann area), B- sumulation from spike train
- regular firing in V1: $C_V = 0.5 \ldots 1$



$$
\begin{aligned}
C_V &= \frac{\sigma}{\mu} \\
pdf^{exp}(x,\lambda) &= \lambda e^{-\lambda x} \\
pdf^{poisson}(x,\lambda) &= \sum_{i=1}^{x} \lambda^i \frac{e^{-\lambda}}{i!}
\end{aligned} \tag{6}
$$

Poisson Spike Train - CODE

```
%% Generation of Poisson spike train with refractoriness
 clear; clf;
 fr_mean=15/1000;       % mean firing rate
%% generating poisson spike train
 lambda=1/fr_mean;                 % inverse firing rate
 ns=1000;                          % number of spikes to be generated
 isi1=-lambda.*log(rand(ns,1)); % generation of expo. distr. ISIs
%% Delete spikes that are within refractory period
 is=0;
 for i=1:ns;
     if rand>exp(-isi1(i)^2/32);
         is=is+1;
         isi(is)=isi1(i);
     end
 end
%% Ploting histogram and caclulating cv
 hist(isi,50);             % Plot histogram of 50 bins
 cv=std(isi)/mean(isi)   % coefficient of variation
```

Sources of noise

- diffuse propagation of neurotransmitter across synaptic cleft
- propagation of the membrane potential along dendtries with varying geometry
- biochemical processes
- probabilistic nature of transmitter release by axonal spikes
- simulation of all these irregularities by INCLUDING NOISE

Noise models I

- Stochastic threshold

$$\vartheta \to \vartheta + \eta^1(t)$$

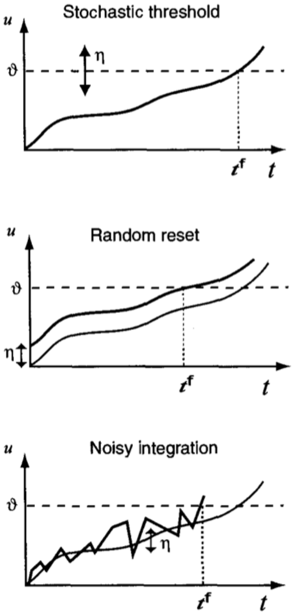
- Random reset

$$v^{\mathrm{res}} \to u^{\mathrm{res}} + \eta^2(t)$$

- Noisy integration

$$\tau_m \frac{dv}{dt} \to -v(t) + RI_{ext} + \eta^3(t) \tag{7}$$

Noise models II



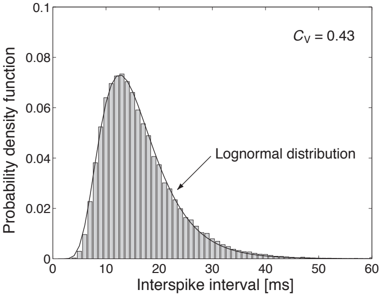
Variability of real neuron

- Model (7): normally distributed current I_{ext}

$$I_{ext} = I_{ext} + \eta, \eta \in N(0,1)$$

- Normal pdf $\rightarrow$ very good approximation considering independent synaptic inputs from many equally distributed neurons
- Simulation: $R\hat{I}_{ext} = 12mV$, $\vartheta = 10mV$
- log normal pdf

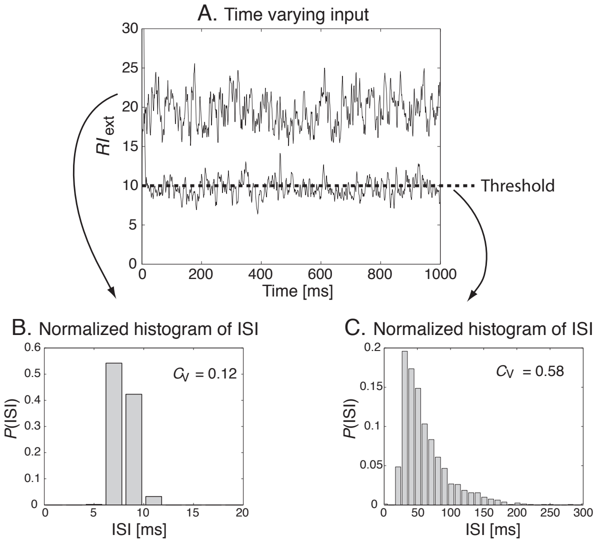
$$pdf^{lognormal}(x;\mu,\sigma) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(\frac{-(\log(x)-\mu)^2}{2\sigma^2}\right.$$



The LIF-neuron noise simulation I

- real neuron with 5000 presynaptic neuron
- 10 % simulation $\rightarrow$ 500 Poisson-distributed spike trains (6) with refractory corrections
- mean firing rate = 20 Hz, after correction 19.3 Hz, refractory constant 2 ms.
- each presynaptic spike $\rightarrow$ EPSP in form of α function (**??**)
- $\omega = 0.5 \rightarrow$ regular firing, $C_V = 0.12$, average rate 118 Hz.
- $\omega = 0.25 \rightarrow$ irregular firing, $C_V = 0.58$, average rate 16 Hz. The $C_V >$ lower bound found in experiments

The LIF-neuron noise simulation II



Further Readings

Wolfgang Maass and Christopher M. Bishop (eds.) (1999), **Pulsed neural networks**, MIT Press.

Wulfram Gerstner (2000), **Population dynamics of spiking neurons: fast transients, asynchronous states, and locking**, in **Neural Computation** 12: 43–89.

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Warren McCulloch and Walter Pitts (1943) **A logical calculus of the ideas immanent in nervous activity**, in**Bulletin of Mathematical Biophysics** 7:115–133.

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Nicolas Brunel and Xiao-Jing Wang, (2001), **Effects of neuromodulation in a cortical network model of working memory dominated by recurrent inhibition**, in **Journal of Computational Neuroscience** 11: 63–85.