Deep Learning (BEV033DLE)
Lecture 6 Weight initialisation, batch normalisation, Resnets

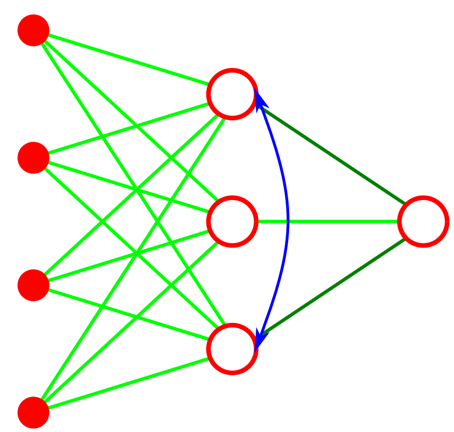
Czech Technical University in Prague

- Weight initialisation
- Batch normalisation
- Residual neural networks
- Transfer learning

Weight initialisation

(1) Initialising all weights and biases with zero is not a good idea. Why?

Side step: symmetries and gradients:

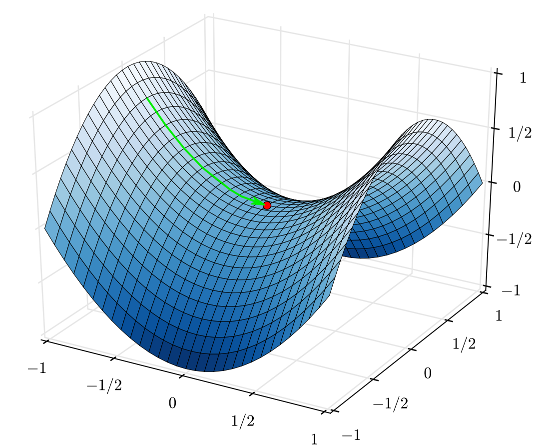


Consider a scalar function $f(w)$ that is invariant to the linear mapping $B\colon \mathbb{R}^n \to \mathbb{R}^n$, i.e. $f(Bw) = f(w)$. Its gradient ∇f has the property

$$\nabla f(Bw) = B^{-T} \nabla f(w),$$

What happens if GD is started from an invariant point $w_0 = Bw_0$ and $B^{-T} = B$ holds?

$$B\big[w_0 - \alpha \nabla f(w_0)\big] = w_0 - \alpha \nabla f(Bw_0) = w_0 - \alpha \nabla f(w_0)$$

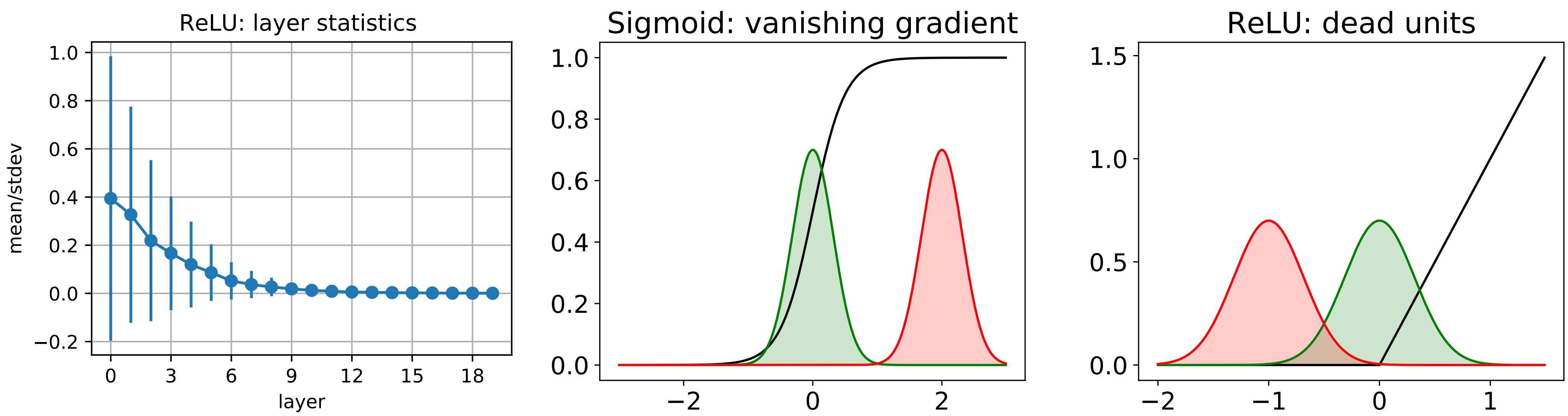


The new point w_1 will be again invariant, i.e. $Bw_1 = w_1$.
We need to break the symmetry!

Weight initialisation

(2) Initialise all weights and biases randomly from a uniform (or normal) distribution.

- o.k. for shallow networks,
- not o.k. for deep networks!



Left: node statistics for the layers of a deep FFN with ReLU units with random inputs, all weights initialised from a normal distribution. Middle and right: this can lead to vanishing/exploding gradients and "dead units" during learning

Weight initialisation

(3) **Proper initialisation:** Initialise weights/biases so that each neuron has activation statistic (over the dataset) with certain mean and variance.

Example 1 (Glorot & Bengio, 2010)**.** Analyse variance of neuron outputs and backprop gradients under the following simplifying assumptions

- Tanh activation function $f(x)$ in linear regime, i,e, $f(x) \approx x$
- Neuron outputs as well as gradient components are i.i.d.

Start from a single neuron $y = w^T x$, $x \in \mathbb{R}^n$. Assume

- x_i are i.i.d. with $\mathbb{E}[x_i] = 0$ and $\mathbb{V}[x_i] = \chi$
- w_i are i.i.d. with $\mathbb{E}[w_i] = 0$ and $\mathbb{V}[w_i] = \omega$

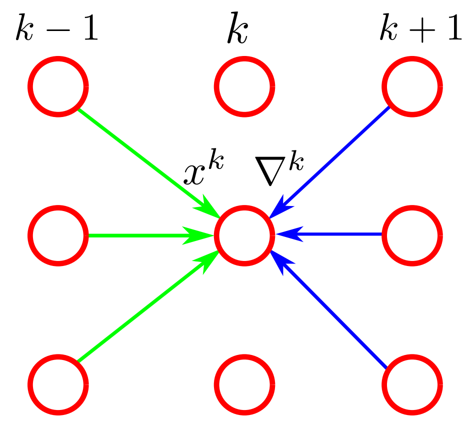
It follows that $\mathbb{E}[y] = 0$ and $\mathbb{V}[y] = n\omega\chi$.

Consider now a feedforward network with Tanh activation and assumptions as above. For layer k with n_k nodes, denote neuron outputs by x^k and gradients by ∇^k. Denote the variance of weights in layer k by ω_k.

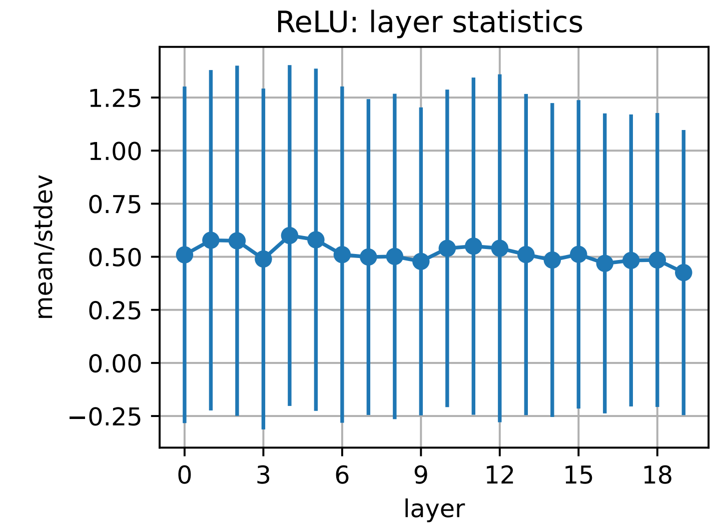
Weight initialisation

Example 1 (cont.).

- forward: $\mathbb{V}[x_i^k] = n_{k-1}\omega_k \mathbb{V}[x_j^{k-1}]$
  We want $\mathbb{V}[x_i^k] \approx \mathbb{V}[x_j^{k-1}]$, i.e. $n_{k-1}\omega_k = 1$.
- backward: $\mathbb{V}[\nabla_i^k] = n_{k+1}\omega_{k+1} \mathbb{V}[\nabla_j^{k+1}]$
  We want $\mathbb{V}[\nabla_i^k] \approx \mathbb{V}[\nabla_j^{k+1}]$, i.e. $n_k\omega_k = 1$
- Compromise: Set $\omega_k = \frac{2}{n_{k-1}+n_k}$. Assuming that the inputs x^0 have zero mean and unit variance, initialise the weights randomly by $w_{ij}^k \sim \mathcal{N}(0, \omega_k)$.



Similar considerations for ReLU activation lead to a different scheme (He et al., 2015). Figure: Node statistics for the layers of the same deep FFN with ReLU units as in Slide 3. But now with a proper weight initialisation.



Batch normalisation

(Joffe & Szegedy, 2015) Motivation:

- Keep control over neuron activation statistics during training
- Alleviate the need of specialised initialisation variants
- Regularise learning & pre-condition gradients

Batch normalisation: Denote by $\mathcal{B} \subset \mathcal{T}^m$ a mini-batch of training examples and by a_i the activation of a network unit $a_i = \sum_j w_{ij} x_j$. Re-parametrise it (stochastically) by using its statistic over mini-batches

$$\mu_{\mathcal{B}} = \mathbb{E}_{\mathcal{B}}[a_i] \quad \sigma^2_{\mathcal{B}} = \mathbb{V}_{\mathcal{B}}[a_i]$$

$$\hat{a}_i = \frac{a_i - \mu_{\mathcal{B}}}{\sqrt{\sigma^2_{\mathcal{B}} + \varepsilon}}$$

$$a_i \leftarrow \gamma \hat{a}_i + \beta \equiv BN_{\gamma,\beta}(a_i)$$

- γ_i, β_i are learnable parameters
- $\mu_{\mathcal{B}}$ and $\sigma_{\mathcal{B}}$ have to be differentiated w.r.t. network parameters
- exponentially weighted averages of $\mu_{\mathcal{B}}$ and $\sigma_{\mathcal{B}}$ are kept during training and used for inference.

Batch normalisation

Technical implementation of batch normalisation in PyTorch: A layer `BatchNorm1d` that

- takes a tensor x with dimension `[batchsize, channels]` on input and returns a tensor y with same dimension on output,
- has learnable parameters γ and β for each channel (init: $\gamma = 1$, $\beta = 0$)
- keeps running averages of the batch statistic $\mu_{\mathcal{B}}$ and $\sigma_{\mathcal{B}}$ for each channel,
- depending on its state (`train`, `eval`) uses either the batch statistics or the saved running averages to compute its outputs.

For convolutional networks: use the layer `BatchNorm2d`, which computes statistics over batchsize and spatial dimensions.

Batch normalisation:

- alleviates the need of special weight initialisation since it implements the scheme (3) discussed above for the first mini batch,
- the neuron outputs for a particular training example depend on the outputs of the other examples in the mini-batch, which in turn is stochastic.
- can be seen as stochastic re-parametrisation of weights and gradient preconditioning

$$w \to \gamma \frac{w}{\sigma_{\mathcal{B}}} \qquad b \to \gamma \frac{(b - \mu_{\mathcal{B}})}{\sigma_{\mathcal{B}}} + \beta$$

Sidestep (biology): Neural Adaptation

Spike frequency range of a biological neuron: 0–500 Hz

Examples of neural adaptation and spike frequency adaptation

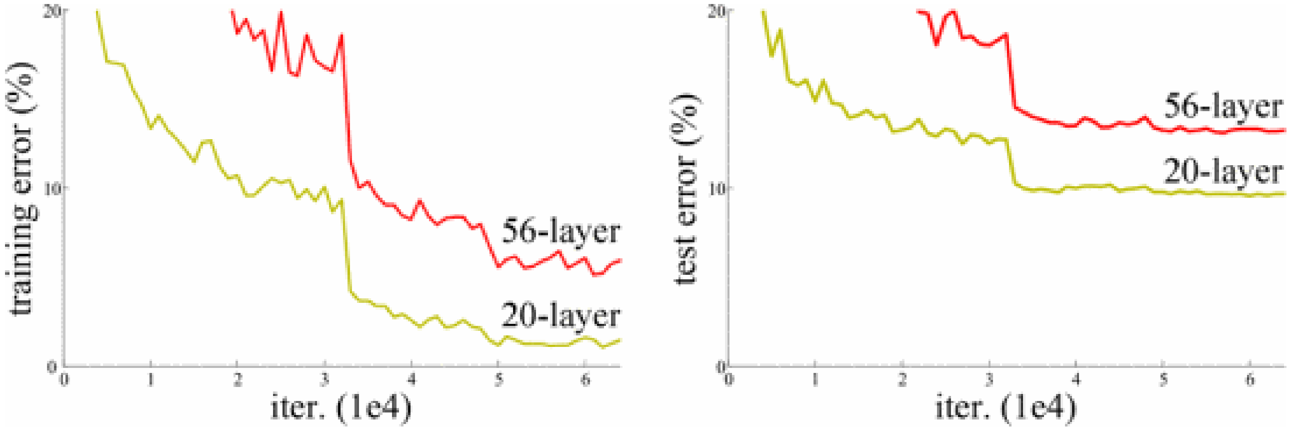
- Sensor adaptation: human eye can function over 9 orders of magnitude of light brightness levels
- Cortical adaptation: neurons in the somatosensory cortex of rodents adapt to periodic stimulation of whiskers (spike frequency decreases over the duration of the stimulus).

Possible mechanisms for spike frequency adaptation:

- short-term synaptic depression: depletion of synaptic vesicles in the pre-synaptic button
- increased spiking threshold: activation of ion channels in the post-synaptic neuron raises the spiking threshold
- lateral and feedback inhibition: diminishes the impact of excitatory inputs over time

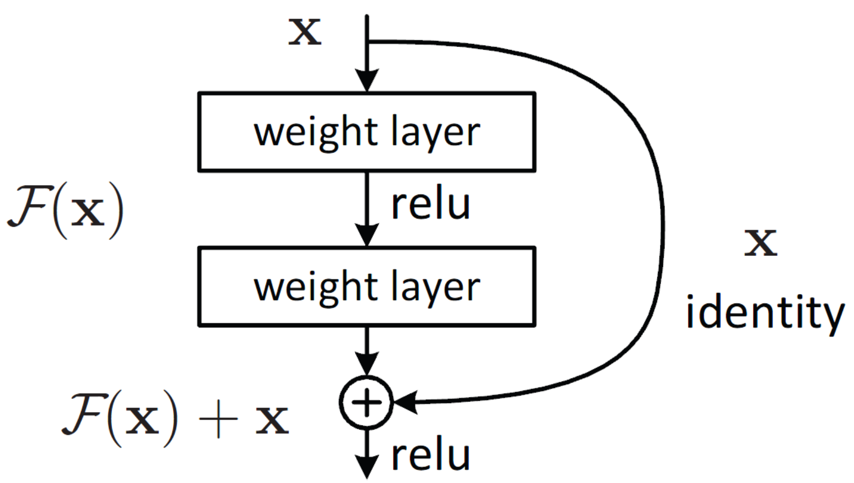
Residual Networks

By using proper weight initialisation/batch normalisation we can learn deep networks with up to 20-30 layers. Can we go for even deeper networks?



Training error and test error on CIFAR-10 with 20-layer and 56-layer "plain" networks.

He et al., CVPR 2016: Yes, by using the architecture of *residual networks*. They introduce *skip connections*.



Residual Networks

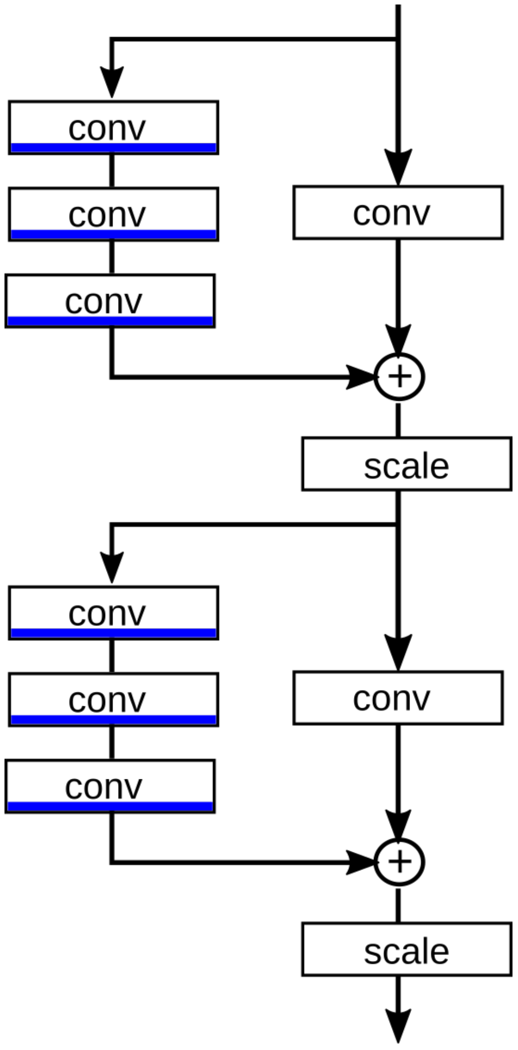
Let us have a different view on residual networks. We have two linked networks:

- a "highway" network with few layers,
- a very deep network which adds "corrections" the former.

This improves trainability of the network. It becomes possible to train networks with 100 and more layers.

Attempts for theoretical explanation:

- analyse the gradients statistic. Resnets have rather uniform distribution of gradients,
- interpret resnets as compositions of "near to identity" mappings and model them by kernels.



Transfer Learning: pre-training & fine-tuning

Transfer learning: pre-training + fine-tuning

- You want to train a predictor for a complex recognition task, but suffer from lack of training data.
- A predictor for a different task has been successfully trained on a large dataset.
- The domains of the two tasks are similar.

We can use the following approach

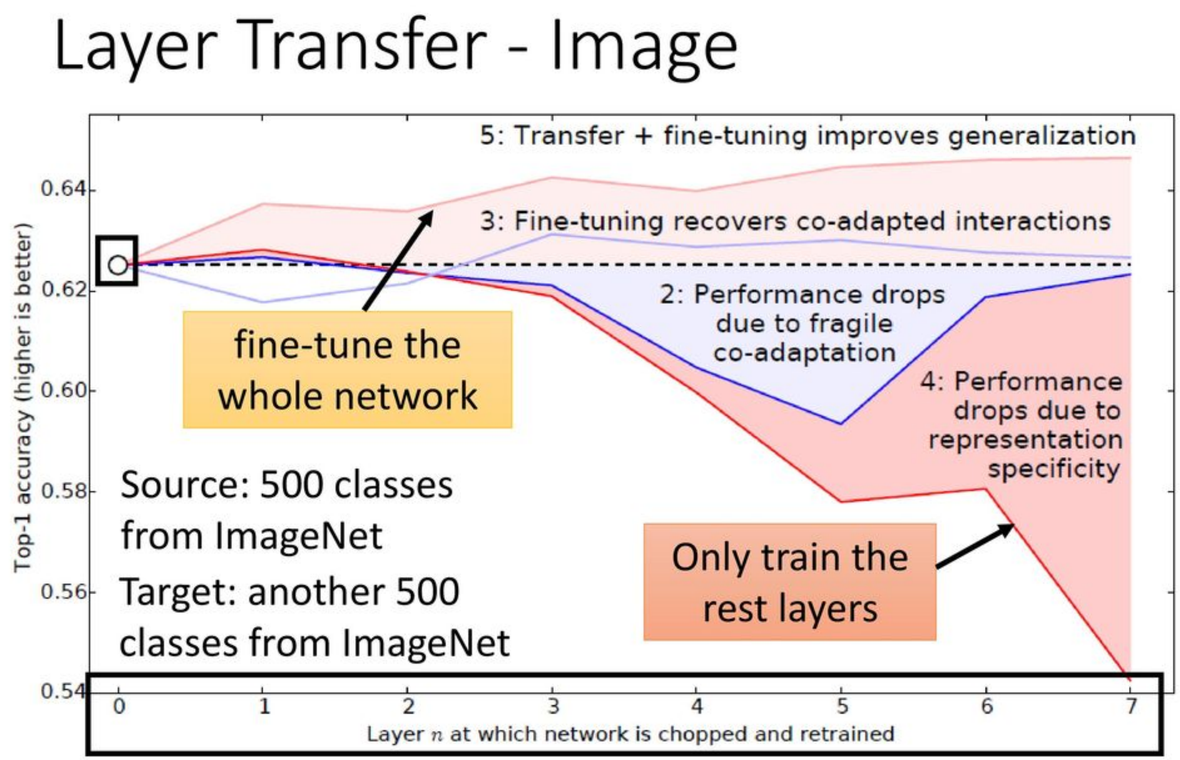
- Use the first layers of the network that implements the predictor for the other task.
- Add your layers on top
- Learn the network on your data, if necessary apply early stopping to prevent overfitting. This can be done in two ways

  (1) freeze the parameters of the transferred layers

  (2) fine-tuning: learn parameters of all layers

Transfer Learning: pre-training & fine-tuning

Example 2 (Yosinski et al., NIPS 2014)**.** Randomly split the 1000 Image-Net classes into two groups with 500 classes: datasets A and B. Learn BnB, BnB^+, AnB and AnB^+ networks. Here: letters indicate the task of the pre-trained/transfer network; n is the number of transferred layers and $+$ indicates the fine-tuning variant.



Jason Yosinski, Jeff Clune, Yoshua Bengio, Hod Lipson, "How transferable are features in deep neural networks?", NIPS, 2014

blue: BnB, BnB^+ red: AnB, AnB^+