Combinatorial Optimization
Lab No. 6
Integer Linear Programming

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Abstract

In this lab, we will further practice problem formulation with Integer Linear Programming. We use it to solve Game of Fivers puzzle and for modeling Power Plants Problem.

1 Game of Fivers

This is a puzzle that is played on a board $n \times n$ with n^2 stones. Each stone has two sides — black and white. In the beginning, each stone is placed on one square facing with white side up. In each turn, the player turns one stone to the other side and with it also its *4-neighborhood*. The goal is to turn all stones to the black face using the minimal number of moves. For humans, boards with sizes $n > 7$ are already challenging ones.

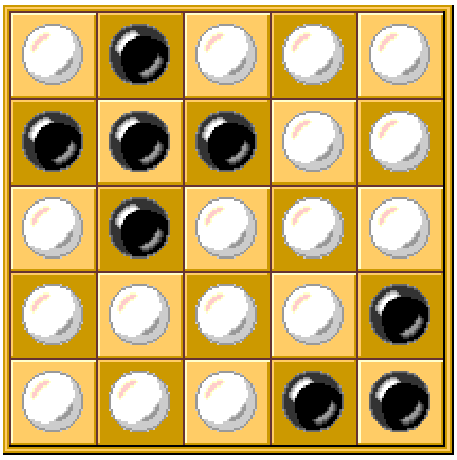

Figure 1: A state of a game in the Game of Fivers.

To formulate this game as an ILP, one needs suitable encoding of the problem — variables, objective and constrains. To do this, one needs to realize following properties of the problem:

1. the order of moves does not matter – the important is what stones will be turned
2. every stone is turned at most once – otherwise there is a way reaching the same state with less moves

3. the last move of the game is characterized by that for each stone, in its 4-neighborhood including the stone itself, an odd number of stones was turned – otherwise we do not end up with all stones facing black

The ILP solves this puzzle *implicitly*, i.e., not giving us a sequence of actions leading to the final state, but it rather encodes necessary conditions for the final state stated by Property 3. Therefore, if ILP solver finds a feasible solution, then it denotes a solution to the puzzle.

A lab exercise: Derive an ILP formulation for Game of Fivers. Then, implement it for general n using Gurobi solver. How large boards can you solve?