

Assume max-flow in  $G$  with zero lower bound:

$$\max \sum_{p \in \bar{P}} x_p$$

$$\text{s.t. } \sum_{p \in \bar{P}: e \in p} x_p \leq u(e) \quad \forall e \in E$$

$$x_p \geq 0 \quad \forall p \in \bar{P}$$

LP duality

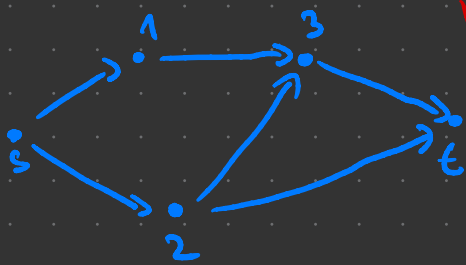


$$\min \sum_{e \in E} u(e) \cdot y_e$$

$$\text{s.t. } \sum_{p \in \bar{P}: e \in p} y_p \geq 1 \quad \forall e \in E \quad (*)$$

$$y_e \geq 0 \quad \forall e \in E$$

$\rightarrow \bar{P}$  is set of all  $s$ - $t$  paths in  $G$ .



$$\bar{P} = \{ (s-1-3-t), (s-2-t), (s-2-3-t) \}$$

$\downarrow$  solve by lazy separation of  $(*)$

- 1) start with a single path (feasible solution)
- 2) find the most violated path  $p \in \bar{P}$  in current solution  $\hat{y}_e$

$$\text{constraints: } \sum_{p \in \bar{P}} y_p - 1 \geq 0 \quad (*)$$

$$\text{Most violated in current solution } \hat{y}_e: \quad p^* := \arg \min_{p \in \bar{P}} \sum_{e \in p} \hat{y}_e - 1$$

$\hookrightarrow$  but this is  $s$ - $t$  shortest path problem in  $G$  with costs  $\hat{y}_e$ !

$\hookrightarrow$  non-negative  $\hat{y}_e$ , use e.g. Dijkstra

$\hookrightarrow$  if you cannot find shortest path lower than 1, then finish.